Math 231/199: Calculus 2 Merit

Worksheet 14

October 23 & 25, 2007

(1) Write out the first 5 terms of the following series:

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{(3k)!}$$

$$\sum_{k=1}^{\infty} \frac{x^k}{2^{k+2}k}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

(2) Write the following power series in Σ-notation:

$$1 + \frac{2}{1!}\,x + \frac{4}{2!}\,x^2 + \frac{8}{3!}\,x^3 + \frac{16}{4!}\,x^4 + \cdots$$

$$x - x^4 + x^7 - x^{10} + x^{13} - x^{16} + \cdots$$

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$$

(3) Write down the Maclaurin series and interval of convergence for the following functions (these will need to be memorized):

$$\sin(x)$$

$$\cos(x)$$

$$\frac{1}{1-x}$$

$$e^x$$

(4) Find the Maclaurin series and its radius of convergence for the following functions:

$$e^{2x}$$

$$\ln(1 + x)$$

(5) Find the Maclaurin series for $1/(1-x)^2$.

(6) Use the Maclaurin series for $1/(1-x)$ to find the Maclaurin series for $1/(1-x)^2$.

(7) Find the Taylor series for $f(x) = \ln(x)$ at the point $c = e$ and find its interval of convergence.

(8) Find the Taylor polynomials $P_n(x)$ with the remainder terms $R_n(x)$ for the following functions at the following points:

$$f(x) = \sqrt{x}, \qquad c = 1, \qquad n = 4$$

$$g(x) = \cos(x), \qquad c = \pi/2, \qquad n = 4.$$

(9) For the following functions, find the remainder term $R_n(x)$ for the function $f(x)$ at $c = 0$. Then show that $\lim_{n\to\infty} R_n(x) = 0$:

$$f(x) = e^{-x}$$

$$f(x) = \sin(x)$$

(10) Approximate $\ln(1.05)$ with the fourth degree Taylor polynomial of $f(x) = \ln(1 + x)$. Estimate the error you make with this approximation. Finally, determine the number of terms of the Taylor series you would need to add to approximate $\ln(1.05)$ to within 10^{-10}.

(11) Use Taylor series to verify the following formulas:

$$\sum_{k=0}^{\infty} \frac{2^k}{k!} = e^2$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k+1}}{(2k+1)!} = 0$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k} = \ln(2)$$

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(12) Use a known Maclaurin series to find the Maclaurin series for the following functions with their radii of convergence:

$$f(x) = e^{-3x}$$

$$f(x) = \frac{e^x - 1}{x}$$

$$f(x) = \sin(x^2)$$

(13) Use Maclaurin series to verify that $\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x)$.

(14) Use Taylor series to show that, for any real number x,

$$e^{ix} = \cos(x) + i\sin(x),$$

where $i = \sqrt{-1}$.

(15) Find the Taylor series for $f(x) = e^x$ with center c.

(16) Find the Maclaurin series for $\cosh(x)$ and $\sinh(x)$. You will need to know that $\cosh(0) = \sinh(0) = 1$.

(17) Show that the Maclaurin series for $(1+x)^r$ is

$$1 + \sum_{k=1}^{\infty} \frac{r(r-1)(r-2)\cdots(r-k+1)}{k!}\, x^r.$$

Find the interval of convergence of this series.