

**Math 231/199: Calculus 2 Merit**

Worksheet 16

November 6, 2007

(1) How many terms of the Maclaurin series for  $\sin(x)$  would you need to sum to approximate  $\sin(1.5)$  to within .00001?

(2) What is the Taylor series for  $\sin(x)$  centered at  $c = \pi/2$ ? How many of its terms would you need to sum to approximate  $\sin(1.5)$  to within .00001?

(3) Based on your answers to the previous two questions, what would be a good guideline to approximating  $f(a)$  with a Taylor series?

(4) Approximate  $e^{-1/10}$  to within  $10^{-8}$ .

(5) Use a series to approximate

$$\int_{-1}^1 e^{-x^2} dx$$

to within  $10^{-6}$ .

(6) Use a series to approximate

$$\int_0^1 \frac{\cos(x) - 1}{x^2} dx$$

to within  $10^{-4}$ .

(7) Why is the method of using series to approximate definite integrals so important?

(8) Try to find

$$\lim_{x \rightarrow 0} \frac{\tan(x) - x}{\sin(x) - x}$$

with L'Hôpital's Rule. Then find the limit using Taylor series.

(9) Find

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x^3}.$$

(10) Find

$$\lim_{x \rightarrow 0^+} \frac{\ln(x)}{1/x}.$$

(11) Show that, for any real number  $x$ ,  $1 + x \leq e^x$ .

(12) Find the Taylor series for  $f(x) = \sqrt[3]{1+x}$  in  $\Sigma$ -notation. Then write out the first five terms of the series.

(13) Find the interior of the interval of convergence for the Taylor series of the last question.

(14) Given your answer to the last question, how could you use the Taylor series for  $\sqrt[3]{1+x}$  to compute  $\sqrt[3]{28}$ ?

(15) Use a Binomial Series to approximate  $\sqrt[3]{28}$  to within .0001.