

Math 231/199: Calculus 2 Merit

Worksheet 18

November 13, 2007

- (1) Find the slope of the given parametric equations at the given points:

$$x = \cos(3t) \quad y = \sin(4t) \quad t = \pi/2$$

$$x = t^3 - t \quad y = t^4 - 5t^2 + 4 \quad t = 1$$

(2) Sketch the graph of the following parametric equation along with the tangent line at the given point. What is the slope of the tangent?

$$x = t^2 - 2 \quad y = t^3 - t \quad (x, y) = (-1, 0)$$

(3) Find all values of t for which the following parametric equations have a horizontal or vertical tangent:

$$x = t^2 - t \quad y = t^4 - 4t^2$$

$$x = 2 \cos(t) + \sin(2t) \quad y = 2 \sin(t) + \cos(2t) \quad 0 \leq t \leq 2\pi$$

(4) Find the vertical and horizontal components of velocity and the speed of the given parametric equations at the indicated point:

$$x = 20t \quad y = 30 - 2t - 16t^2 \quad t = 2$$

$$x = 2 \cos(2t) + \sin(5t) \quad y = 2 \sin(2t) + \cos(5t) \quad t = 0$$

(5) Sketch the parametric equation

$$x = \cos(2t) \quad y = \sin^2(t).$$

Estimate the slope at the point $t = 0$.

(6) For the parametric equation of the last question, show that $x'(0) = y'(0) = 0$ but that the slope at $t = 0$ exists (has a real number value).

(7) Find the area enclosed by the following parametric equations:

$$x = t \sin(t) \quad y = t \cos(t) \quad 0 \leq t \leq \pi/2$$

$$x = t^3 - 4t \quad y = t^4 - 1 \quad -1 \leq t \leq 1$$

(8) Find the area enclosed by the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

by integrating $\int y \, dx$.

(9) Find the area enclosed by the ellipse of the last question using parametric equations.

(10) Suppose that an object follows the path

$$x = \sin(4t) \quad y = -\cos(4t).$$

Show that the speed of the object is constant and that the tangent line is always perpendicular to the line from the object to the origin.