

Math 231/199: Calculus 2 Merit

Worksheet 4

September 4, 2007

(1) Let θ be a first-quadrant angle with $\tan(\theta) = y$. Determine the values of the other five trigonometric functions of θ .

(2) Find

$$\int \frac{1}{\sqrt{x^2 - 4x}} dx.$$

(3) Find

$$\int \frac{1}{x^2 \sqrt{16 - x^2}} dx.$$

(4) Find $\int x^3 \sqrt{x^2 - 1} dx$.

(5) Find

$$\int \frac{x^2}{\sqrt{x^2 + 9}} dx.$$

(6) Find $\int x^2\sqrt{4-x^2} dx$.

(7) Find

$$\int_0^1 \frac{1}{\sqrt{x(1-x)}} dx.$$

(8) What is the equation for the top half of a circle with radius r centered at the origin?

Use this equation to find the area of a half-circle with radius r .

(9) Hyperbolic trigonometry is to the hyperbola what trigonometry is to the circle. It is defined in terms of two functions $\sinh(x)$ and $\cosh(x)$, related by the identity $\cosh^2(x) = 1 + \sinh^2(x)$. All ratios ($\tanh(x)$, $\coth(x)$, $\operatorname{sech}(x)$, and $\operatorname{csch}(x)$) are defined as in ordinary trigonometry. Given that $\frac{d}{dx} \sinh(x) = \cosh(x)$ and $\frac{d}{dx} \cosh(x) = \sinh(x)$, use a hyperbolic trigonometric substitution to find $\int \frac{1}{4-x^2} dx$.