Math 231/199: Calculus 2 Merit

Worksheet 9

October 2, 2007

(1) Identify the first term and common ratio of the following geometric series:

$$\sum_{k=0}^{\infty} \frac{2}{4^k}$$

$$\sum_{k=2}^{\infty} 2^{k/2}$$

$$\sum_{k=1}^{\infty} \left(\frac{1}{5}\right)^{k-1}$$

$$\sum_{k=4}^{\infty} (-1)^k \frac{3^{k/2}}{4^{k-1}}$$

(2) Find the first six partial sums of the following series:

$$\sum_{k=1}^{\infty} \frac{1}{3^k}$$

$$\sum_{k=2}^{\infty} \frac{1}{k}$$

$$\sum_{k=0}^{\infty} \frac{3}{4}$$

$$\sum_{k=1}^{\infty} \sin(\pi k)$$

(3) Find a formula for the nth partial sum S_n of the following series, and use it to determine if the series converges or diverges:

$$\sum_{k=1}^{\infty} \frac{1}{2^k}$$

$$\sum_{k=1}^{\infty} \left(\frac{1}{k+1} - \frac{1}{k} \right)$$

$$\sum_{k=1}^{\infty} 1$$

(4) Determine if the following series converge or diverge. If they converge, find their value:

$$\sum_{k=0}^{\infty} 5\left(\frac{1}{3}\right)^k$$

$$\sum_{k=0}^{\infty} \left(\frac{5}{3}\right)^k$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k k^3}{k^2+1}$$

$$\sum_{k=0}^{\infty} \frac{4k}{k+2}$$

$$\sum_{k=1}^{\infty} \frac{9}{k(k+3)}$$

$$\sum_{k=1}^{\infty} \frac{2}{k}$$

$$\sum_{k=3}^{\infty} \frac{10}{k(k+2)}$$

$$\sum_{k=2}^{\infty} \sin(k)$$

(5) Determine if the following series converge or diverge. If they converge, find their value:

$$\sum_{k=0}^{\infty} \left(\frac{1}{2^k} - \frac{1}{k+1} \right)$$

$$\sum_{k=1}^{\infty} \left(\frac{2}{3^k} + \frac{1}{2^k} \right)$$

(6) Prove that if $\sum_{k=1}^{\infty} a_k = L$, then for all positive integers m, $\sum_{k=m}^{\infty} a_k$ converges, and determine its value.

(7) Consider the series $\sum_{k=2}^{\infty} \frac{1}{k}$. Split the terms into blocks $B_1, B_2, \ldots$ such that B_k contains the reciprocal of each number between $2^{k-1}+1$ and 2^k. Show that the sum of the terms in B_k is at least $1/2$ for all k. What does this say about convergence/divergence of the harmonic series?

(8) Show that the series $1 - 1 + 1 - 1 + \cdots$ diverges by looking at the sequence of partial sums. Can you think of another way to show this series diverges?

(9) Write $.45454545\overline{45}$ as a series. What does this series sum to?

(10) Suppose \$100,00 of counterfeit money is introduced into the economy. Each time the money is used 25% is identified as counterfeit and removed from circulation. Determine the total amount of counterfeit money successfully used in transactions.

(11) Give an example where $\sum_{k=0}^{\infty} a_k$ and $\sum_{k=0}^{\infty} b_k$ diverge but $\sum_{k=0}^{\infty}(a_k + b_k)$ converges.

(12) Prove that the sum of a convergent geometric series $\sum_{k=0}^{\infty} r^k$ is always greater than $1/2$.

(13) Prove that the series $\sum_{k=1}^{\infty}(a_k - a_{k-1})$ converges if and only if the sequence $\{a_k\}_{k=0}^{\infty}$ converges.

(14) The *Cantor set* is defined through the following procedure: start with the interval $[0, 1]$ and remove the middle third (that is, $(1/3, 2/3)$), then remove the middle thirds of the remaining intervals ($(1/9, 2/9)$ and $(7/9, 8/9)$), and continue to remove the middle third of every interval ad infinitum. How much "length" does the Cantor set have?

(15) Is it surprising, given your answer to the last part, that the Cantor set contains as many points as the interval $[0, 1]$?

(16) Two bicyclists are initially 40 miles apart and begin to bike toward each other at 10 miles per hour each. At the moment they start, a fly, who flies at 20 miles per hour, starts at one of the bicyclists and flies toward the other. When it reaches the other bike, it turns around and heads toward the other biker. The fly continues to fly back and forth between the two bikers until they meet. Write a series to determine how far the fly travels in total.

(17) How long does it take the bikers in the last question to reach each other? Given that, how far would a fly going 20 miles per hour travel in that time?