How to Write Mathematics

1 Introduction

The goal of writing mathematics is the same as all types of expository writing: to clearly and completely express an idea. Writing mathematics is also similar in flavor to other kinds of writing; one uses words, sentences, and paragraphs.

Often one does not do mathematics for oneself. In a course, one does math for the instructor. At a job, one does math for the boss. Therefore, being able to express the mathematical ideas is as important as knowing how to solve problems. Also, clearly explaining the ideas ensure you really understand them.

Several tips for writing mathematics include:

1) Let the reader know what you are doing: What problem are you trying to solve? What are you trying to show and how will this help solve the problem?

2) Let the reader know what you are using: What theorems, rules, and general principles are you using and how are they being applied?

3) Let the reader know what you have done: Recap what you have already done, and make it clear when you are finished. You can symbolically indicate the end of a solution by writing QED or drawing a box □ at the end of the line. You can also use terms such as "...and so we are done" or "..., which completes the solution".

4) Define all your symbols: Don't use symbols your reader doesn't understand. Explain what your functions are. For example, "Let $h(t)$ be the height of the projectile at time t".

5) Be clear: Don't confuse the reader. Often one can be clearer by numbering equations (simply write a number to the right of it) and referring to them later. For example, "By combining equations (1) and (3), we see that..." or "By the argument of the second paragraph,...". Also, defining an extra

variable or function can save effort in referring to certain quantities later on.

2 Bad Mathematics

What is wrong with the following solutions?

PROBLEM: A water tank is shaped like an inverted cone with height and base radius each 5 meters. Starting at time $t = 0$, the tank is filled at a rate of 1 m^3/s. If the tank is empty when $t = 0$, what is the height of the water at time t (in terms of t)?

SOLUTION:

$$t = \tfrac{\pi}{3} h(t)^3$$

$$\sqrt[3]{\tfrac{3t}{\pi}}$$

PROBLEM: Find the derivative of $\cos(\sin^2(x))$.

SOLUTION: $$-2\cos(x)\sin(x)\sin(\sin^2(x))$$

PROBLEM: Find $\int \frac{\ln t}{t}\,\mathrm{d}t$.

SOLUTION:

$$\begin{matrix} \ln t & & \tfrac{1}{t} \\ \tfrac{1}{t} & & \ln t \end{matrix}$$

$$\ln^2(t) - \int \frac{\ln t}{t}\,\mathrm{d}t$$

$$\frac{\ln^2 t}{2}$$

PROBLEM: A rectangular box is three times as long as it is deep. Its depth is the square of its height. The volume of the box is 3072 cubic inches. How long is it?

SOLUTION:

$$3x^{5/2} = 3072$$
$$x = 16$$

3 Good Mathematics

What about these solutions is better than those of the last section?

PROBLEM: A water tank is shaped like an inverted cone with height and base radius each 5 meters. Starting at time $t = 0$, the tank is filled at a rate of 1 m^3/s. If the tank is empty when $t = 0$, what is the height of the water at time t (in terms of t)?

SOLUTION: Let $h(t)$ be the height of the water at time t. Notice that at time t, t cubic meters of water are in the tank.

Recall the volume of a cone with height and base radius equal to x is $(\pi/3)x^3$. Therefore, the volume of water in the tank at time t is $(\pi/3)h(t)^3$.

Combining what we know about the volume, we see

$$t = \frac{\pi}{3}h(t)^3,$$

and so

$$h(t) = \sqrt[3]{\frac{3t}{\pi}}.$$

PROBLEM: Find the derivative of $\cos(\sin^2(x))$.

SOLUTION: Let $Q(x) = \cos(\sin^2(x))$.

By the Chain Rule, the derivative of $f(g(h(x)))$ is

$$(g(h(x)))'f'(g(h(x))) = h'(x)g'(h(x))f'(g(h(x))).$$

Let $f(x) = \cos(x)$, $g(x) = x^2$, and $h(x) = \sin(x)$, so that $Q(x) = f(g(h(x)))$. Then $f'(x) = -\sin(x)$, $g'(x) = 2x$, and $h'(x) = \cos(x)$. Plugging these in to the above formula, we get

$$Q'(x) = (\cos x)(2\sin x)(-\sin(\sin^2 x)) = -2\cos(x)\sin(x)\sin(\sin^2(x)).$$

QED

PROBLEM: Find $\int \frac{\ln t}{t}\,\mathrm{d}t$.

SOLUTION: Let $I = \int \frac{\ln t}{t}\,\mathrm{d}t$.

Recall Integration by Parts tells us $\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u$. If we set $u = \ln t$ and $\mathrm{d}v = \mathrm{d}t/t$, then $\mathrm{d}u = \mathrm{d}t/t$ and $v = \ln t$. Plugging these in to the formula, we get

$$I = (\ln t)(\ln t) - \int \frac{\ln t}{t}\,\mathrm{d}t = (\ln t)^2 - I.$$

Adding I to both sides and dividing by 2, we see

$$I = \frac{(\ln t)^2}{2},$$

so we are done.

PROBLEM: A rectangular box is three times as long as it is deep. Its depth is the square of its height. The volume of the box is 3072 cubic inches. How long is it?

SOLUTION: Let x be the depth of the box (in inches). Then the length is $3x$ and the height is $\sqrt{x}$, so the volume is $3x^{5/2}$. Since this equals 3072, we know $x^{5/2} = 1024$, or $x = 16$.

Since the length of the box is $3x$, the box is 48 inches long.