SOLUTIONS FOR HOMEWORK 1

2.1.20. Yes.

Recall that the elements of $P(A)$ are subsets of A. Then $A = \cup_{X \in P(A)} X$. If $P(A) = P(B)$, then $A = \cup_{X \in P(A)} X = \cup_{Y \in P(B)} Y = B$.

Alternative solution. $c \in P(A)$ has exactly one element if and only if $c = \{a\}$, for some $a \in A$. Suppose, for the sake of contradiction, that x belongs to A, but not to B. then $\{x\} \in P(A)$, but $\{x\} \notin P(B)$.

2.2.14. First, show that $(A \cap B) \cup (A - B) = A$.

It follows immediately from the definition that $A - B = A \cap \overline{B}$. To show that $A = (A \cap B) \cup (A - B)$, use the distributive law for the union of sets:

$$(A \cap B) \cup (A \cap \overline{B}) = A \cap (B \cup \overline{B}) = A \cap U = A.$$

In our case, $A = (A \cap B) \cup (A - B) = \{1, 5, 7, 8\} \cup \{3, 6, 9\} = \{1, 3, 5, 6, 7, 8, 9\}$. Similarly, $A = (A \cap B) \cup (B - A) = \{2, 3, 6, 9, 10\}$.

2.2.16. (e) We shall prove two inclusions: $A \cup (B - A) \subseteq A \cup B$ and $A \cup (B - A) \supseteq A \cup B$.

If $x \in A \cup (B - A)$, then x belongs to either A, or to $B - A$, hence to B. Either way, $x \in A \cup B$. This proves the inclusion $A \cup (B - A) \subseteq A \cup B$.

Conversely, consider $x \in A \cup B$. If $x \in A$, then $x \in A \cup (B - A)$. If $x \notin A$, then $x \in B - A$, hence $x \in A \cup (B - A)$. This proves the inclusion $A \cup (B - A) \supseteq A \cup B$.

Alternative solution. $B - A = B \cap \overline{A}$, hence, by the distributive law,

$$A \cup (B - A) = A \cup (B \cap \overline{A}) = (A \cup B) \cap (A \cup \overline{A}) = (A \cup B) \cap U = A \cup B$$

(here, U denotes the universal set).

2.2.18. (d) Suppose, for the sake of contradiction, that $x \in (A - C) \cap (C - B)$. In other words, x belongs to both $A - C$ and $C - B$. The former condition implies $x \notin C$. However, if $x \in (C - B)$, then $x \in C$. Hence, a contradiction.

(e) As noted before, $(B - A) \cup (C - A) = (B \cap \overline{A}) \cup (C \cap \overline{A})$. By the distributive law for unions of sets,

$$(B \cap \overline{A}) \cup (C \cap \overline{A}) = (B \cup C) \cap \overline{A} = (B \cup C) - A.$$

2.2.24. We have to show that (i) $(A - B) - C \subseteq (A - C) - (B - C)$, and (ii) $(A - B) - C \supseteq (A - C) - (B - C)$

(i) $x \in (A - B) - C$ iff $x \in (A - B)$, but $x \notin C$. Using the definition of $A - B$, we see that the above is equivalent to $x \in A$, $x \notin B$, and $x \notin C$. Therefore, $x \in (A - C)$, but $x \notin (B - C)$ (since $x \notin B$ to begin with). Thus, $x \in (A - C) - (B - C)$. This establishes $(A - B) - C \subseteq (A - C) - (B - C)$.

(ii) Suppose $x \in (A - C) - (B - C)$. Then $x \in (A - C)$, and $x \notin (B - C)$. The former condition means that $x \in A$, and $x \notin C$. The latter condition states that one of the following cases holds: (a) $x \notin B$, or (b) $x \in B \cap C$. However, (b) is impossible, since we already know that $x \notin C$. Therefore, x belongs to A, and doesn't belong to either B or C. Hence, $x \in (A - B) - C$ iff $x \in (A - B)$. This establishes the inclusion $(A - B) - C \supseteq (A - C) - (B - C)$.

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