SOLUTIONS FOR QUIZ 5

A bakery has bagels of four types: plain, poppy seed, sesame seed, and raisin. How many ways are there to make a selection of 11 bagels in such a way that at least two plain bagels are included? All bagels of one type are assumed to be indistinguishable.

Answer: $\binom{9+4-1}{9} = \binom{12}{9} = \binom{12}{3}$.

Denote the number of plain, poppy seed, sesame seed, and raisin bagels in our selection by i_1, i_2, i_3, and i_4, respectively. We are looking for number of solutions of the equation $11 = i_1+i_2+i_3+i_4$, where $i_1, \ldots, i_4$ are non-negative integers, and moreover, $i_1 \geq 2$. Define j_s $(1 \leq s \leq 4)$ by setting $j_1 = i_1-2$, and $j_s = i_s$ for $s \in \{2, 3, 4\}$. Then j_1, j_2, j_3, and j_4 are non-negative integers, and $j_1 + j_2 + j_3 + j_4 = 9$. There is a one-to-one correspondence between the solutions of this equation, and the appropriate solutions of $11 = i_1 + i_2 + i_3 + i_4$.

Thus, we have to find the number of 4-tuples (j_1, j_2, j_3, j_4) of non-negative integers, satisfying $j_1 + j_2 + j_3 + j_4 = 9$. This is the same as the number of 9-compositions of 4 elements with repetitions, that is, $\binom{9+4-1}{9} = \binom{12}{9} = \binom{12}{3}$.

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