MATH 213 MIDTERM 2: PRACTICE PROBLEMS

The test will be given on **Wednesday, November 19**. It will be based on Homeworks 6-10, covering the material from Sections 6.1-7.5 (probability, recurrence relations, and inclusion-exclusion).

In preparing for the test, practice solving the problems from this list. In addition, take a look at the homework (at least one problem on the midterm will come directly from the homework), and at the examples given in the textbook.

1. For each of the following recurrence relations, find an explicit formula for a_n.

(a) $a_n = a_{n-2} - 2a_{n-1}$, $a_0 = 2$, $a_1 = 1$.

(b) $a_n = 2a_{n-1} + 3a_{n-2} + 4(-1)^n$, $a_0 = 1$, $a_1 = -4$.

2. Suppose a random variable X attains only four values – 0, 1, 2, and 3, with $p(X = 0) = 1/2$, $p(X = 1) = p(X = 2) = p(X = 3) = 1/6$. Compute the expected value and the variance of X.

3. A fair coin is tossed four times. Consider the events E and F: E occurs when Heads appears exactly twice, and F occurs when Heads appears exactly once on the first two tosses. Are the events E and F independent?

4. An urn contains 98 red balls and 2 green balls. Balls are drawn from the urn without repetition, until a green ball is encountered. Denote the number of drawings performed by X. Compute $p(X = k)$ $(1 \leq k \leq 99)$.

5. Among the students taking a certain course, 25% are Biology majors, 25% are Physics majors, and 50% are Math majors (no multiple majors). 80% of Biology majors, 60% of Physic majors, and 40% of Math majors pass this course. What is the probability that a student passing the course is a Math major?

6. Suppose the events E and F are such that $p(F) = 1/3$, $p(E|F) = 4/5$, and $p(E|\overline{F}) = 1/5$. Compute $p(E)$.

7. Suppose the events A and B satisfy $p(A|B) < p(A)$. Prove that $p(B|A) < p(B)$.

8. The 52 cards of the standard deck are dealt to four players in such a way that each player get 13 cards. What is the probability that each player gets an ace?

9. How many integers between 1 and 1000 are divisible by 3 and 7, but not by 5?

10. Suppose $X_1, X_2, \ldots, X_8$ are Bernoulli random variables, such that $p(X_i = 1) = 1/4$ for each i, and $p(X_i = X_j = 1) = 1/14$ whenever $i \neq j$. Find the expected value and the variance of $X = X_1 + \ldots + X_8$.

Hint. Recall that $\mathrm{V}(\sum_{i=1}^{8} X_i) = \sum_{i=1}^{8} \mathrm{V}(X_i) + \sum_{i \neq j} \mathrm{Cov}(X_i, X_j)$. See the class notes, or Exercise 38 from Section 6.4, for the definition of the covariance.

11. There are 5 sets, consisting of 100 elements each. Pairwise intersections of these sets contain 25 elements each, the intersection of any three contain 10 elements, and no element belongs to more than three of the five sets. Compute the number of elements in the union of the five sets.

To: the syllabus, the main page of the course, the solutions.