

## 1 Section 1.4

- 13 Name the columns of  $A$   $\mathbf{v}_1, \mathbf{v}_2$ . For  $\mathbf{u}$  to be in the span of the columns of  $A$  means that there exist constants  $x_1, x_2$  such that  $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \mathbf{u}$ . By Theorem 3 solving this equation for  $x_1, x_2$  is equivalent to finding solutions of the systems of equations whose associated augmented matrix is  $[A \ \mathbf{u}]$ . Now,  $[A \ \mathbf{u}]$  is the matrix

$$\begin{pmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{pmatrix}$$

After computing the row reduced form (not shown here) we see that there is one free variable, namely  $x_2$ , so  $\mathbf{u}$  is in  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ .

- 15 To solve the equation  $Ax = b$  Theorem 3 tells us that we must row reduce the augmented matrix

$$\begin{pmatrix} 2 & -1 & b_1 \\ -6 & 3 & b_2 \end{pmatrix}$$

First multiply the top row by 3 and add it to the bottom row to get

$$\begin{pmatrix} 2 & -1 & b_1 \\ 0 & 0 & 3b_1 + b_2 \end{pmatrix}$$

We see by Theorem 2 from section 1.2 that this has solutions if and only if  $3b_1 + b_2 = 0$ , or equivalently  $b_2 = -3b_1$ . This answers both questions posed in the problem.

- 18 By Theorem 4, the columns of  $B$  span  $\mathbb{R}^4$  if and only if  $B$  has a pivot in every row. We can see by the reduced echelon form of  $B$  that it does NOT have a leading 1 in the last row. Therefore, Theorem 4 says that the columns of  $B$  do NOT span  $\mathbb{R}^4$ . Further, using Theorem 4, since 4(c) is false, 4(a) is false as well, so  $Bx = y$  does not have a solution for each  $y$  in  $\mathbb{R}^4$ .
- 34 We are given a  $3 \times 3$  matrix  $A$  and a *particular* vector  $\mathbf{b} \in \mathbb{R}^3$ . Since  $A\mathbf{x} = \mathbf{b}$  has a unique solution, we know that the augmented matrix has

a pivot in every column except for the last one. Therefore  $A$  must have a pivot in every column and, because it's a square matrix, in every row. Theorem 4 now implies that the columns of  $A$  span  $\mathbb{R}^3$ .

## 2 Section 1.7

- 6 To determine if the columns of a matrix  $B$  are linearly independent we must check if the equation  $A\mathbf{x} = \mathbf{0}$  has *only* the trivial solution. So row reduce the augmented matrix  $[B \ 0]$  to echelon form and notice that there is a pivot in every column and thus all variables are basic. We may conclude that there is a unique solution (Theorem 2) and since  $\mathbf{0}$  is a solution, then it must be the only solution. Therefore the columns of  $B$  are linearly independent.
- 15, 18 Notice that we are given 4 vectors in  $\mathbb{R}^2$ . Theorem 8 now implies that this set must be linearly dependent.
- 20 Notice that since the zero vector is one of the vectors, Theorem 9 implies that the set must be linearly dependent.