Math 220(GE1) Practice Exam 1 Solutions
Please notice: The solutions here are mostly a sketch to give you an idea for what the answer should be.

1) Find the natural domain and range of the following functions:

(a) $f(x) = 2\sin(x-1)$

Domain: $(-\infty, \infty)$. Range: $[-2, 2]$.

(b) $f(x) = e^{x^2}$

Domain: $(-\infty, \infty)$ Range: $[1, \infty)$.

(c) $f(x) = \ln(x^2)$.

Domain: $(-\infty, 0) \cup (0, \infty)$. Range: $(-\infty, \infty)$.

2) Use the graph of $y = \ln(x)$ to sketch the following graphs.

(a) $y = \ln(1-x)$

(b) $y = 2 + \ln(3x)$

You can use your calculator to check your graphs for both of these.

3) (a) State the limit definition of the derivative (any of the four forms is acceptable).

For example:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Let $f(x) = x^2 - 7$. Use the limit definition of the derivative to find $f'(x)$. (You will *not* get credit for using the Power Rule here.)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2 - 7 - (x^2 - 7)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 7 - x^2 + 7}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2}{h} \\
&= \lim_{h\to 0} 2x + h \\
&= 2x
\end{aligned}$$

4) Find an equation for the line tangent to the curve

$$y = 2x^{\frac{3}{2}} + \frac{5}{x}$$

at $x = 1$.

First we figure out that $y'(x) = 3x^{1/2} - \frac{5}{x^2}$. Since $y(1) = 7$, and the slope of the tangent line to $y(x)$ at $x = 1$ is given by $y'(1) = -2$, the equation is: $y - 7 = -2(x - 1)$.

5) The graph of $f'(x)$, the *derivative* of $f(x)$, is shown below. Answer the following questions, and **briefly** justify your answer.

(a) On which intervals is f increasing?

Where f' is positive, so $(-5, -2) \cup (1, 4)$.

(b) At which values of x does f have a local maximum?

By the first derivative test, f has a local maximum at $x = -2, 4$.

(c) On which intervals is f concave down?

Where f' is decreasing, so $(-5, -4) \cup (-3, -1) \cup (2, 2.5) \cup (3.5, 5)$.

(d) At which values of x does f have an inflection point?

Where f changes concavity. This occurs at $x = -4, -3, -1, 2, 2.5, 3.5$

(e) At which values of x does f' have a stationary point?

At the points where f' has a horizontal tangent, so at $x = -4, -3, -1, 2, 2.5, 3.5$.

(f) At which values of x does f have a stationary point?

At the points where $f'(x) = 0$, so at $x = -2, -1, 4$.

6) Let $f(x) = \begin{cases} x^2 + ax - 3 & \text{if } x < 2 \\ x - 4 & \text{if } x \geqslant 2 \end{cases}$

(a) Evaluate $\lim_{x \to 2^+} f(x)$.

This limit is equal to -2.

(b) Evaluate $\lim_{x \to 2^-} f(x)$.

This limit is equal to $1 + 2a$.

(c) Find a value for a that makes f continuous at $x = 2$.

Solving $1 + 2a = -2$ for a we see that $a = -\frac{3}{2}$.

(d) If we use the value of a from part (c), does f'(2) exist?

No, since to the left of $x = 2$ the derivative "wants to be" 2.5 and to the right of $x = 2$ it "wants to be" 1.

7) (a) Give an example of a continuous function which is not differentiable at $x = 1$.

$f(x) = |x - 1|$

(b) Sketch a function f such that $f''(x) > 0$ and $f(x) > 0$ for all x in $(-\infty, \infty)$.

e^x.

(c) Sketch a function g such that $g(x) > 0$ and $g'(x) > 0$ for all x.

e^x.

8) Decide if the following statements are true or false. You do not need to justify your answers.

(a) Every continuous function is periodic.

False.

(b) Every continuous function is differentiable at all points of its domain.

False.

(c) If $\lim_{x \to 5} f(x) = 3$ and $\lim_{x \to 5} g(x) = 3$ then $\lim_{x \to 5} (\frac{f(x)}{g(x)}) = 1$.

True.

(d) All local maxima or minima occur at stationary points of a function.

True.

(e) A function is concave up at $x = a$ if $f''(a) > 0$.

True.

(f) If $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

True.