Math 220(GE1) Practice Exam 1

Justify your answers!

1) Find the natural domain and range of the following functions:

(a) $f(x) = 2\sin(x-1)$

(b) $f(x) = e^{x^2}$

(c) $f(x) = \ln(x^2)$.

2) Use the graph of $y = \ln(x)$ to sketch the following graphs.

(a) $y = \ln(1-x)$

(b) $y = 2 + \ln(3x)$

3) (a) State the limit definition of the derivative (any of the four forms is acceptable).

(b) Let $f(x) = x^2 - 7$. Use the limit definition of the derivative to find $f'(x)$. (You will *not* get credit for using the Power Rule here.)

4) Find an equation for the line tangent to the curve

$$y = 2x^{\frac{3}{2}} + \frac{5}{x}$$

at $x = 1$.

5) The graph of $f'(x)$, the *derivative* of $f(x)$, is shown below. Answer the following questions, and **briefly** justify your answer.

(a) On which intervals is f increasing?

(b) At which values of x does f have a local maximum?

(c) On which intervals is f concave down?

(d) At which values of x does f have an inflection point?

(e) At which values of x does f' have a stationary point?

(f) At which values of x does f have a stationary point?

6) Let $f(x) = \begin{cases} x^2 + ax - 3 & \text{if } x < 2 \\ x - 4 & \text{if } x \geqslant 2 \end{cases}$

(a) Evaluate $\lim_{x \to 2^+} f(x)$.

(b) Evaluate $\lim_{x \to 2^-} f(x)$.

(c) Find a value for a that makes f continuous at $x = 2$.

(d) If we use the value of a from part (c), does f'(2) exist?

7) (a) Give an example of a continuous function which is not differentiable at $x = 1$.

(b) Sketch a function f such that $f''(x) > 0$ and $f(x) > 0$ for all x in $(-\infty, \infty)$.

(c) Sketch a function g such that $g(x) > 0$ and $g'(x) > 0$ for all x.

8) Decide if the following statements are true or false. You do not need to justify your answers.

(a) Every continuous function is periodic.

(b) Every continuous function is differentiable at all points of its domain.

(c) If $\lim_{x \to 5} f(x) = 3$ and $\lim_{x \to 5} g(x) = 3$ then $\lim_{x \to 5} (\frac{f(x)}{g(x)}) = 1$.

(d) All local maxima or minima occur at stationary points of a function.

(e) A function is concave up at $x = a$ if $f''(a) > 0$.

(f) If $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.