Final Study guide for Math 220 GE1.

Chapters 1-3

Basics of functions

- Trigonometric functions: Know the graphs and behaviour of $\sin(x)$ and $\cos(x)$, including their domains and ranges and the values they take at the "celebrated" angles. You should also know the definitions of the other trig functions (e.g., $\tan(x) = \frac{\sin(x)}{\cos(x)}$).
- Logs and Exponentials: Know the graphs and behaviour of e^x, $\ln(x)$, and related functionsm including their domains and ranges. You should know the relation between values of logs and the corresponding exponential functions.
- Inverse functions: You should know how to find the formulas for and domains of inverse functions, as well as their derivatives. Remember: $f^{-1}(x)$ is NOT $1/f(x)$!

Basics of Derivatives

- Know the limit definition of the derivative.
- Know the relationships between the behavior of the function and its derivatives (e.g., $f''(x)$ positive means $f(x)$ is concave up).
- Know how to classify stationary points using the First and Second Derivative Tests. You may be asked to state one or both.
- Know the derivative formulas for $x^k, \sin(x), \cos(x), b^x, \log_b(x)$, and inverse trig functions.
- Be able to use the product rule, quotient rule, and chain rule.
- Be able to perform implicit differentiation.
- Know how to find the line tangent to a curve at a point.

Differential Equations

- You should know how to solve the elementary differential equations we covered in class. Specifically: those solveable by antidifferentiating both sides, those in the form $y' = ky$, and those in the form $y'' = -ky$. You should know the general formula for the solutions of these last two.
- You should know how to use initial values to find exact solutions.
- You should know what an IVP is and be able to write one down from a

word problem, as on the mastery exam.

Chapter 4

Limits (4.2)

Pretty much all types of limits are fair game. In particular:
- Be sure you know how to use L'Hopital's rule (including when you're allowed to use it).
- Know how to handle other sorts of indeterminant forms - things that look like $0 * \infty$, $1/0$, etc.
- You will not need to know the Squeeze Principle.
Practice problems: 4.2: 9-30, 36-39, 41, 52-78, 82

Optimization (4.3)

- Know what it means for something to be a *critical point* of a function how to find critical points.
- Be able to find the max or min of a function on a closed interval.- You should know how to find the max or min on other sorts of intervals.
- Be able to solve simple constrained optimization problems.
Practice problems: 4.3: 3, 4, 9-13, 18, 22, 24-28

Parametric Equations (4.4)

You should know how to graph simple parametric equations. Remember that the allowed values given for t matter!
Practice problems: 4.4: 1-5, 11, 12

Related Rates (4.5)

You should be able to use the "yoga" of related rates problems from lab 8 to solve related rates problems.
Practice problems: 4.5: 4-10, 16, 17, 19, 20

Taylor Polynomials (4.7)

- Be able to find the nth order Taylor Polynomial about x_0, given either a function and the base point x_0, or the values of the function and its derivatives

at the base point.
- Understand how Taylor polynomials are good approximations of the original function close to x_0, and be able to use them to approximate a function's value.
Practice problems: 4.7: 8-10, 16-18, 21, 23, 32, 34

Fun with theorems! (4.8 and 4.9)

- Know the statements of the following theorems: IVT, EVT, MVT, Rolle's Theorem, Racetrack Principle.
- Understand how to use theorems to solve simple problems, as in homework and lab. You may be asked to determine if the hypothesis and conclusion of a theorem hold for a particular function, or to provide an example of a function that satisfies the conclusion of a theorem, but not the hypothesis.
Practice problems: 4.8: 7-18, 31-34, 39; 4.9: 15-22, 24-26, 28

Chapter 5

Basics of Integrals

- Know the definition of the integral as signed area as given on page 303.
- Know how to write the signed areas of regions between a curve and the x-axis as integrals.
- Know how to write the areas of regions bounded by curves as integrals. (Remember, these are always positive areas!)
- Know how to use Theorem 1 on page 306 and the definition on page 311 for swapping the limits of integration.
- Know the statement of the "Bounding an Integral" Fact on page 307 and be able to use it.
- Know the definition of the average value of a function given on page 309.
- Know Theorems 2 and 3 on page 322, Theorem 4 on page 325 and the "Fact" on page 327.
- Be able to state at least one form of the Fundamental Theorem of calculus.
- Be comfortable with an integral $F(x) = \int_a^x f(t)dt$ as an antiderivative of $f(x)$. Given $f(x)$, you should be able to answer all of the usual questions about where $F(x)$ is increasing, decreasing, concave up or down, has stationary points, etc etc.
Practice problems: 5.1 #2-4, 33-36

Practice problems: 5.3 #1-63.

Antidifferentiation

- Know *all* the antiderivatives in the table on page 333, as well as the antiderivative of $\ln(x)$. You will not be responsible for others in the back of the book.
- Know how to use u-substitution to evaluate definite and indefinite integrals, including how to change the endpoints of definite integrals.
- Always remember you need to specify d-something when integrating!
- Likewise, remember that a definite integral gives you a number. An indefinite integral gives you a function, so don't forget that $+C$.

Practice problems: 5.4 #1-74

Riemann Sums

- Know the definition of a Riemann sum on page 352
- Know the definition of an integral as a limit of Riemann sums on page 353. You should be able to state this "Riemann Sum definition of the integral".
- Know how to compute left, right, and midpoint Riemann sums and trapazoid sum approximations from a picture, table, or formula.
- Be familiar with the basics of Sigma notation. Some questions will specify that you need to use Sigma notation for your sums.
- Know how to write the general formula for left, right, and midpoint Riemann sums in Sigma notation.
- Be able to write left, right, and midpoint Riemann sums for specific functions in Sigma notation.