

Mastery Exam Practice 2 Solutions

- (1) Compute the derivative of  $f(x) = \frac{2}{x}$  using the definition of the derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2x - 2(x+h)}{x(x+h)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2h}{x(x+h)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2}{x(x+h)} = -\frac{2}{x^2} \end{aligned}$$

- (2) Find an equation of the tangent line to the curve  $y = 3x + \sin(x)$  at the point where  $x = 0$ .

$y'(x) = 3 + \cos(x)$ .  $y'(0) = 3 + \cos(0) = 4$ . At  $x = 0$ ,  $y(0) = 3(0) + \sin(0) = 0$ , so the line with slope 4 going through  $(0, 0)$  is given by  $y = 4x$ .

- (3) (a) If  $10^{1.69} = 49$ , what is an approximate value of  $\log_{10}(7)$ ?  
 $10^{1.69} = 49 = 7^2$ . Thus  $10^{.845} = 7$ , so  $\log_{10}(7) = .845$ .

- (b) Find  $\tan(\frac{\pi}{6})$ .  
 $\frac{1}{\sqrt{3}}$

- (4) Let  $f(x) = 2 \ln(x+1) - 3$ .

- (a) Find a formula for  $f^{-1}$ .

$$\begin{aligned} x &= 2 \ln(y+1) - 3 \\ (x+3)/2 &= \ln(y+1) \\ e^{(x+3)/2} &= y+1 \\ y &= e^{(x+3)/2} - 1 \end{aligned}$$

(b) What is the domain of  $f^{-1}$ ? Explain.

The domain is all real numbers.

(5) Find the derivative of  $f(x) = e^\pi + \cos(2x) \ln(x)$ .

$$f'(x) = 0 + -2 \sin(2x) \ln(x) + \cos(2x) \frac{1}{x}$$

(6) Find the derivative of  $f(x) = 5^{(x^4 + \cos(x))} + x^6$ .

$$f'(x) = \ln(5)5^{(x^4 + \cos(x))}(4x^3 - \sin(x)) + 6x^5.$$

(7) Find the derivative of  $f(x) = \frac{e^{15x} + 12x}{\sin(x)}$ .

$$\frac{(15e^{15x} + 12)(\sin(x)) - \cos(x)(e^{15x} + 12x)}{\sin^2(x)}$$

(8) Sketch a graph of  $g(x) = 1 + 3 \cos(3x)$ . Indicate the scale on both the  $x$  and  $y$  axes.

Check on your calculator.

(9) Suppose that  $f$  is a function with derivative  $f'(x) = (x^2 - 1)e^x$ .

(a) Determine the set of points  $x$  on which  $f$  is increasing.

The roots of  $f'(x)$  are 1 and  $-1$ . For  $x < -1$ ,  $f'(x) > 0$ . For  $x$  in  $(-1, 1)$ ,  $f'(x) < 0$ , and for  $x > 1$ ,  $f'(x) > 0$ . Thus  $f$  is increasing on  $(-\infty, -1) \cup (1, \infty)$ .

(b) Find all the stationary points of the function  $f$ .

$$x = -1, 1.$$

(c) Which of the stationary points are maximum, minimum, or neither? Explain.

By the first derivative test,  $x = -1$  is a local max and  $x = 1$  is a local min.

(10) Find the equation of the tangent line to  $\sin(xy) = x + y$  at the point  $(0, 0)$ .

Implicitly differentiate:

$$\begin{aligned}\cos(xy)(y + xy') &= 1 + y' \\ \cos(xy)y + \cos(xy)xy' &= 1 + y'\end{aligned}$$

Solve for  $y'$ :

$$\begin{aligned}y' - \cos(xy)xy' &= \cos(xy)y - 1 \\y'(1 - \cos(xy)x) &= \cos(xy)y - 1 \\y' &= \frac{\cos(xy)y - 1}{1 - \cos(xy)x}\end{aligned}$$

At  $(0,0)$ ,  $y' = -1/1 = -1$ , so the equation of the tangent line is  $y = -x$ .

- (11) If a culture of mold doubles in size every 8 hours and there is 5 grams of mold at noon, find an equation for the grams of mold in terms of hours past noon.

Let  $w(t)$  denote the grams of mold in terms of hours past noon. We know that  $w(t) = Ae^{kt}$  for some constants  $A$  and some constants  $t$ . Since  $w(0) = 5$ , we have that  $A = 5$ . We know that  $w(8) = 10$ , but  $w(8) = 5e^{8k} = 10$ ,  $k = \ln(2)/8$ .

Thus  $w(t) = 5e^{\frac{\ln(2)}{8}t}$ .