Practice final

1) Find the following limits

(a) $\lim_{x \to 0} \frac{\sin(2x)}{x}$

(b) $\lim_{x \to 3} \frac{x^2-4}{x^2-2}$

(c) $\lim_{x \to \infty} \frac{x}{e^x}$

2) Find the derivative of each of the following functions

(a) $f(x) = \ln(x^2) + 3x^2$

(b) $g(x) = \frac{x^2}{\sin(x)}$.

(c) $h(x) = e^{3x}\sin(x^2)$.

(d) $w(t) = \sqrt{\frac{t^2+3}{\tan(t)}}$

3) Evaluate each of the following integrals

(a) $\int\limits_0^1 x^2 + \sin(\pi x) + \frac{1}{x+1}\, dx$

(b) $\int xe^{x^2}\, dx$

(c) $\int \tan x\, dx$

(d) $\int\limits_{-\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} x\cos(3x^2)\, dx$

4) Let $f(x) = (x^2 - 1)^{1/3}$.

(a) Find the maximum and minimum values f takes on the interval $[-2, 1]$.

(b) Find upper and lower bounds for $\int_{-2}^{1} f(t)\, dt$. (If you were unable to finish part (a), you may leave your answer in terms of m and M, but you must explain what they mean.)

5) Find the third-order Taylor polynomial for $\sin(x)$ centered about $x_0 = \frac{\pi}{2}$.

6) Let $F(x) = \int_0^x f(t)\,dt$, where $f(x) = (x-3)(x+4)$. Answer the following questions about $F(x)$, justifying your answers.

(a) Where is F decreasing?

(b) Where is F concave up?

(c) What are the stationary points of F?

(d) Where does F have local minima?

(e) Where does F have inflection points?

(f) Suppose $G(x) = \int_1^x f(t)\,dt$. How are the graphs of F and G related? Be specific.

7) Does $f(x) = \cos(3x) + \sin(3x)$ solve the IVP $y'' = -9y$, $y(0) = 5$? Justify your answer.

8) A giant vat of molasses has developed a leak, losing 1 cm^3 of molasses a minute. The leaked molasses forms a circular puddle 1 cm thick. How fast is the radius of the puddle increasing when the puddle's volume is 9π cm^2?

9) Let $f(x) = x^3$. Approximate $\int_{-1}^{1} f(t)\,dt$ by the limit of *midpoint* Riemann sums. Express your answer in sigma notation.

10) Let $f(x) = |x^2 - 16|^2$. Answer the following questions, giving justification for your answers.

(a) Does f satisfy the hypothesis of the Intermediate Value Theorem on $[-5, 5]$?

(b) Does f satisfy the hypothesis of the Mean Value Theorem on $[-5, 5]$?

(c) Does f satisfy the conclusion of the Intermediate Value Theorem on $[-5, 5]$

(d) Does f satisfy the conclusion of the Mean Value Theorem on $[-5, 5]$? Find c if it exists.

11) State the Extreme Value Theorem.

12) Find the equation for the line tangent to the curve $y^2 + x^2 = 1 + \sin(x)$ at the point $(0, 1)$.