Math 220, Spring 2007

Here are some comments on a few group-work problems.

Even and Odd functions

1 Let's begin with the problem that asked: *Suppose that* $q(t) = (t+1)(t-2)^2$. *Determine if* $q(t)$ *is even, odd, or neither.*

To answer this question correctly - with a proper understanding of a solution - we must take a look at how mathematics interacts with language. The best way to go about that is by analogy:

Suppose that I said: "Everyone in Memorial Stadium is wearing an orange shirt." How would you check if this statement is true? How would you check if this statement is false? In both cases you would visit memorial stadium and look at everyone's shirts. If you find even one person not wearing an orange shirt, then I am wrong. If you find that everyone is wearing an orange shirt, then I am correct.

Now let $g(t) = (t+1)(t-2)^2$ with the domain $(-\infty, \infty)$. Is $g(t)$ even, odd, or neither?

To answer this we must first look at the definitions. Just like we all know what an orange shirt looks like, by now we all know that a function $g(t)$ is odd if for every t in its domain, $g(-t) = -g(t)$. $g(t)$ is even if $g(-t) = g(t)$. To answer the question I must either examine all the points in g's domain to make sure that they fit one of the rules, or I must find at least one point which doesn't fit either rule. The former is done with symbols, while the latter is done with specific numerical examples.

Warning: thinking about graphs of even and odd functions as having certain symmetries is very useful! That's how you can get a clue as to what the answer should be. In this case, the graph of $g(t)$ shows us that this function is probably neither odd nor even.

But talking about the graph is insufficient. In the analogy, you found a specific person not wearing an orange shirt. In our example, you should find a point that rules out $g(t)$ being odd, and a point that rules out $g(t)$ being even. You could get this information from the graph (make sure that you know how!), but a simple guess of $t = -1$ does the trick: $g(-1) = 0$ while $-g(1) = -2$. Therefore, since $g(-1) \neq -g(1)$, $g(t)$ is not odd. Since $g(-1) \neq g(1)$, $g(t)$ is not even.

2 Moving on, here's a harder question that you recently discussed: *Let* $q(x) = ax^2 + bx + c$ *where* $a, b,$ *and* c *are constants. Suppose that* q *is an even function. Show that* $b = 0$ *must be true.*

A lot of confusion arose from a misunderstanding of the question. Let's break it up by sentences: The first sentence *"Let ..."* simply says "You're given a function $q(x)$ that has a certain formula." The next sentence *"Suppose that* q *is an even function"* says that you are *assuming* that q is even. In other words, q satisfies the property that $q(-x) = q(x)$. Finally, *"Show that* $b = 0$*"* is what you're being asked to do.

I've dissected this question in detail because as Calculus students you are not very familiar with the intricate play between mathematics and language. After all, it takes mathematicians many years to fully develop this skill, albeit in more advanced settings. You may recall from other courses that when faced with a word problem you often were asked to write what you're given and what you want to find. In mathematics, "suppose" is one of the words often used to list what you're given.

In this problem we're given an even function $q(x) = ax^2 + bx + c$ and we must show that $b = 0$. Remember, you can only use what you're given! In this case, the only information available to us is that q is even. Thus, $q(-x) = q(x)$. Well, we have a formula for q, so let's use it: $q(-x) = a(-x)^2 + b(-x) + c = ax^2 - bx + c$; $q(x) = ax^2 + bx + c$. Since q is even, $q(-x) = q(x)$, which after using the formula gives: $ax^2 - bx + c = ax^2 + bx + c$. Adding $-ax^2$ and $-c$ to both sides we obtain $-bx = bx$. What does this imply about b? Since we are allowed to set $x = 1$, we have that $-b = b$. But the only number equal to its negative is 0, so $b = 0$ must be true.

The second part of this question asked what can be said about a, b, and c if q is odd. Again, we're being told to assume that q is odd. Let's try an approach similar to the one above: If q is odd, then for all x, $q(-x) = -q(x)$. Using our formula we get that $ax^2 - bx + c = -ax^2 - bx - c$. Adding bx to both sides gives $ax^2 + c = -ax^2 - c$. Plugging in $x = 0$ gives $c = -c$ - thus $c = 0$ must be true. If $c = 0$, then we're left with $ax^2 = -ax^2$, and plugging in $x = 1$ gives $-a = a$, implying that $a = 0$ must be true.

So far we've discovered that if q is odd, then $a = 0$ and $c = 0$ must be true. What about b? If $q(x) = bx$, then $q(-x) = -bx = -q(x)$. This means that for any value b that we pick (including 0), $q(x) = bx$ will be an odd function.