

**Solutions to: 3.3: 9-14**

- (9) Let's say that Curve 1 is  $x^2 + xy + y^2 = 1$  and Curve 2 is  $x^2 - xy + y^2 = 1$ . To find  $\frac{dy}{dx}$  for Curve 1, we differentiate both sides:  $2x + xy' + y + 2yy' = 0$  and solve for  $y'$  to get  $y' = -\frac{2x+y}{x+2y}$ . For Curve 2, using the same method, we get that  $y' = \frac{-2x+y}{-x+2y}$ .
- (10) Looking at the picture of both curves, we notice that one of them will have a tangent line with positive slope at  $(0, 1)$  while the other will have a tangent line with the negative slope at  $(0, 1)$ . Evaluating  $y'$  of both curves at this point we can figure out which one is which.
- (12) The curve  $x^2 + xy + y^2 = 1$  has  $y' = 0$  if and only if  $2x + y = 0$ . This line,  $y = -2x$  intersects the curve at the point  $(1/\sqrt{3}, -2/\sqrt{3})$ . By symmetry, the other points with horizontal tangent lines are  $(-1/\sqrt{3}, 2/\sqrt{3})$ ,  $(1/\sqrt{3}, 2/\sqrt{3})$ , and  $(-1/\sqrt{3}, -2/\sqrt{3})$ .
- (13) For the curve  $x^2 + xy + y^2 = 1$ , the tangent line is vertical wherever  $y'$  is undefined. From the equation of  $y'$  we see that this is precisely when  $y = -x/2$ . Thus the curve is vertical at the points  $(2, \sqrt{3}, -1/\sqrt{3})$  and  $(-2/\sqrt{3}, 1/\sqrt{3})$ . For the other curve, those points are  $(2/\sqrt{3}, 1/\sqrt{3})$  and  $(-2/\sqrt{3}, -1/\sqrt{3})$ .
- (14) The formulas for  $y'$  show that if  $y = x$ , then  $y' = -1$  for both curves. Since the line  $y = x$  has slope 1, it follows that the line  $y = x$  is perpendicular to both curves at the points of intersection.

