Solutions to: 4.2: 67, 69, 70, 77

(67)

$$\begin{aligned}
\lim_{x\to 1}\frac{\ln(x)}{x^2-x} &= && \textit{we're in the 0/0 indeterminate form}\\
&= \lim_{x\to 1}\frac{\frac{1}{x}}{2x-1} && \textit{by l'Hopital's rule}\\
&= \lim_{x\to 1}\frac{1}{2x^2-x} && \textit{no need to simplify, but it's easy to see the limit now}\\
&= 1
\end{aligned}$$

(69)

$$\begin{aligned}
\lim_{x\to 1}\frac{\sin(\pi x)}{x^2-1} &= && \textit{we're in the 0/0 indeterminate form}\\
&= \lim_{x\to 1}\frac{\pi\cos(\pi x)}{2x} && \textit{by l'Hopital's rule}\\
&= -\frac{\pi}{2} && \textit{Just plug in 1 to the above expression}
\end{aligned}$$

(70)

$$\begin{aligned}
\lim_{x\to 0}\frac{1-x^2-e^{-x^2}}{x^4} &= && \textit{we're in the 0/0 indeterminate form}\\
&= \lim_{x\to 0}\frac{-2x-(e^{-x^2}(-2x))}{4x^3} && \textit{by l'Hopital's rule}\\
&= \lim_{x\to 0}\frac{2x(-1+e^{-x^2})}{4x^3}\\
&= \lim_{x\to 0}\frac{-1+e^{-x^2}}{2x^2} && \textit{simplified, this is in indeterminate form 0/0}\\
&= \lim_{x\to 0}\frac{-e^{-x^2}(-2x)}{4x} && \textit{by l'Hopital's rule}\\
&= \lim_{x\to 0}\frac{-e^{-x^2}}{2}\\
&= -\frac{1}{2}
\end{aligned}$$

(77)

$$\begin{aligned}
\lim_{w\to 0^+} w\ln(w)^2 &= && \textit{this is in the } 0\cdot\infty \text{form}\\
&= \lim_{w\to 0^+}\frac{ln(w)^2}{\frac{1}{w}} && \textit{rewriting into } \frac{\infty}{\infty} \text{ form}\\
&= \lim_{w\to 0^+}\frac{2\ln(w)\frac{1}{w}}{-\frac{1}{w^2}} && \textit{by l'Hopital's rule}\\
&= \lim_{w\to 0^+} -\frac{2\ln(w)}{\frac{1}{w}} && \textit{still in } \frac{\infty}{\infty} \textit{ form}\\
&= \lim_{w\to 0^+} -\frac{\frac{2}{w}}{\frac{-1}{w^2}} && \textit{apply l'Hopital's rule}\\
&= \lim_{w\to 0^+} 2w && \textit{simplify}\\
&= 0
\end{aligned}$$