Solutions to homework due on Friday, 3/30

(4.6: 4a) We want to use Newton's method to find an approximate value of $10^{1/3}$. That is, we want to find a root of $x^3 - 100$. Beginning with $x_0 = 4.5$, $x_1 = 4.5 - \frac{(4.5)^3-100}{3(4.5)^2-100} = 4.64609$. Continuing, $x_2 = 4.65169$ and $x_3 = 4.65159$. The last two agree to three decimal places, so we're done.

(4.6: 9) (a) When $x_0 = 2$, $x_1 = 1.2500000$, $x_2 = 1.02500000$, $x_3 = 1.000304878$, and $x_4 = 1.000000046$. They are accurate to 0, 1, 3, and 7 decimal places respectively.

(b) Newton's method finds $x = 1$ if $x_0 > 0$; it finds $x = -1$ if $x_0 < 0$. It fails if $x_0 = 0$.

(4.6: 16) Look at the graph and use Newton's method. Use the fact that $h(-x) = h(x)$ to your advantage.

(4.9: 8) The parabola $x^2 - 1$ will have two roots and only one stationary point.

(4.9: 9) Not possible: Rolle's theorem implies that any such function will have to have a stationary point on the intervals (a, b) and (b, c) where a, b, c are the roots of f.

(4.9: 10) $f(x) = x^3 - x + 2$ has one root and two stationary points.

(4.9: 11) $f(x) = \sin(x)$ has infinitely many roots and stationary points.

(4.9: 12) This is really an IVP: If $f'(x) = -1$, then $f(x) = -x + C$, and since $f(0) = 137$, we get that $f(x) = -x + 137$.

(4.9: 15) f does not satisfy the MVT hypothesis on $[-1, 2]$ since it is not differentiable at $x = 0$.

(4.9: 16) Since $f'(x) = 1$ or $f'(x) = -1$, and $(f(2) - f(-1))/(2 - -1) = 1/3$, there does not exist a number c in $(-1, 3)$ such that $f'(c) = 1/3$.

(4.9: 17) Yes, since f is differentiable (and therefore continuous) on $[1, 2]$, it satisfies the MVT hypothesis for that interval.

(4.9: 18) By the above, f has to satisfy the conclusion of the MVT since it satisfies the hypothesis for the MVT. On $[1, 2]$, $((f(2) - f(1))/(2 - 1) = 1$, and any c in $(1, 2)$ will do.

(4.7: 1) Use theorem 6 to note that $f(0) = 1$, $f'(0) = 1$, $f''(0) = 2$, $f'''(0) = 6$, $f^{(4)}(0) = 120$, $f^{(5)}(0) = 96$, $f^{(6)}(0) = 0$, and $f^{(131)}(0) = 0$.

(4.7: 2) Use theorem 6 and realize that the only non-zero derivative is $f^{(5)}(1) = 120$.

(4.7: 8) If $f(x) = \sqrt{x}$, then $f'(x) = 1/(2\sqrt{x})$ and $f''(x) = -1/(4x^{3/2})$. At $x_0 = 100$, $f(100) = 10$, and $f'(100) = 1/20$, and $f''(100) = -1/4000$. Thus $T_1(x) = 10+1/20(x-100)$ and $T_2(x) = 10+1/20(x-100)+-1/8000(x-100)$. $T_1(121) = 11.05$ and $T_2(121) = 10.9949$. Since $\sqrt{121} = 11$, T_1 overestimates by 0.05 and T_2 underestimates by 0.0051.

(4.7: 17) Linear: $T_1(x) = 1 + x$. Quadratic: $T_2(x) = 1 + x + x^2/2$.

(4.7: 18) Linear: $T_1(x) = x$ and $T_2(x) = x$.