Solutions to homework from sections 5.3 and 5.4, extra credit problems

(5.3: 32) Since $G(x) = \int_a^b \sin(t)dt$, by the FTC we get that $G'(x) = \sin(x)$. From this we see that the slope of the tangent line to $G(x)$ at $x = \sqrt{\pi}$ is $\sin(\sqrt{\pi})$ and the rest is easy.

(5.3: 48) By the FTC, $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$, and also by the FTC, $\int_a^x \left[\frac{d}{dt}f(t)\right] dt = f(x) - f(a)$. Thus, unless $f(a) = 0$, the two are not equal.

(5.4: 36) $\int \frac{x^3}{1+x^2}dx$. Let $u = 1 + x^2$. Then $du = 2xdx$, and thus $\frac{du}{2} = xdx$. Also, $u-1 = x^2$ by our definition of u. Thus, $x^3 = x^2x = (u-1)\frac{du}{2}$. Substituting we get $\int \frac{(u-1)}{2u}du$, and taking the $\frac{1}{2}$ out of the integral and simplifying the inside we get $.5\int 1 - \frac{1}{u}du$, whose anti-derivative is $.5u + .5\ln(u) + C$ (since u is always positive by our definition, the ln has no absolute values). Plugging back in for u we get $.5(1 + x^2) + .5\ln(1 + x^2) + C$.

(5.4: 40) $\int \frac{x}{\sqrt{1+x^2}}dx$. Let $u = 1+x^2$. Then $du = 2xdx$, and $\frac{du}{2} = xdx$. Substituting we get $\int \frac{1}{2\sqrt{2}}du$. After anti-differentiating the final answer is $\frac{1+x^2}{+}C$.

(5.4: 52) $\int \sec(x)\tan(x)dx$. Let $u = \sec(x)$. Then $du = \sec(x)\tan(x)dx$. Substituting we get $\int du = u + C = \sec(x) + C$.

(5.4: 60) $\int x^2 \sec^2(x^3)dx$. Let $u = x^3$. Then $du = 3x^2dx$ and $\frac{du}{3} = x^2dx$. Substituting we get $\int \frac{\sec^2(u)}{3}du$. Then the final answer is $\frac{1}{3}\tan(x^3) + C$.

(5.4: 12) Let $u = 3x^3$. From here it's easy.

(5.4: 62) $\int x^4 \sqrt[3]{x^5 + 6}dx$. Let $u = x^5 + 6$. Then $du = 5x^4dx$, and $\frac{du}{4} = x^4dx$. Substituting we get $\int \frac{\sqrt[3]{u}}{5}du$. The final answer then is $\frac{3(x^5+6)^{4/3}}{20} + C$.

(5.4: 63) $\int \frac{1+\sqrt{x})^3}{\sqrt{x}}dx$. Let $u = 1 + \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}dx$, and $2du = \frac{1}{\sqrt{x}}dx$. Substituting we get $\int 2u^3du$, and after antidifferentiating and plugging in for u the final answer is $.5(1 + \sqrt{x})^4 + C$.

(5.4: 51) $\int \tan(x)dx$. Let $u = \cos(x)$. Then $du = -\sin(x)dx$, and $-du = \sin(x)$. Substituting we get $\int -\frac{1}{u}du$. Antidifferentiating and plugging in for u the final answer is $-\ln(|\cos(x)|) + C$.

(5.4: 55) $\int \frac{5x}{3x^2+4}dx$. Let $u = 3x^2 + 4$. Then $du = 6xdx$, and substituting we get $\frac{5}{6}\int \frac{du}{u} = \ln(|u|) + C = \frac{5}{6}\ln(3x^2 + 4) + C$. (Note that since $3x^2 + 4$ is always positive, we drop the absolute value).

Extra Credit Problems: Due Wednesday, 4/25 Solutions must be written clearly. A correct solution is worth 5 homework points.

(1) Let

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Show that f is differentiable at 0 but f' is not continuous at 0.

(2) During the final round of a game show you're presented with three doors. Behind one of them is a prize, and behind the other two are goats. You're first asked to pick one of the three doors. After you make your choice of a door, the host opens another door with a goat behind it, and asks if you want to pick the other unopened door. Does switching doors increase your chances of winning the prize? Calculate the precise probabilities in support of your answer.