

(59) If f is such that $\int_0^3 f(x)dx = -1$, then if f is even, $f(-x) = f(x)$, so

$\int_{-3}^3 f(x)dx = \int_{-3}^0 f(x)dx + \int_0^3 f(x)dx$. We're given that the latter integral is equal to -1 , and to evaluate the former integral we let $u = -x$. Then $du = -dx$, and $\int_{-3}^0 f(x)dx = \int_3^0 -f(u)du = \int_0^3 f(u)du = -1$. Then the sum is -2 as claimed. Another way to see it is that an even function is symmetric about the y -axis, so the signed area under f on $[0, 3]$ is equal to the signed area under f on $[-3, 0]$.

If f is an odd function, then $\int_{-3}^3 f(x)dx = \int_{-3}^0 f(x)dx + \int_0^3 f(x)dx$, but the former integral (using the same method as above) turns out to be equal to 1 , and thus the sum is 0 . Again, symmetry is another way to see that this is the case.

(60) We know that on $[0, \pi]$, $\sin(x) \geq 0$ and $x \geq 0$, and that the minimum value of the two functions is 0 . Thus, the signed area under the graph of $x \sin(x)$ on $[0, \pi]$ will be at least 0 . Also, $x \sin(x)$ achieves its maximum value at $\frac{\pi}{2}$, which means that the integral will be bounded by $\frac{\pi}{2}\pi = \frac{\pi^2}{2}$.

(80) Let $f(x) = \sqrt{4+x}$. Then the right-hand Riemann sum approximation for $f(x)$ on the interval $[0, 5]$ is equal to $\frac{5}{n} \sum_{k=1}^n \sqrt{4+5k/n}$, and taking the limit as $n \rightarrow \infty$, we obtain the value $\int_0^5 \sqrt{4+x}dx$. Using substitution we can evaluate this integral, and it equals $\frac{38}{3}$. Notice that we also could have used the function $f(x) = \sqrt{x}$ and the interval $[4, 9]$, as it would have given us the exact same Riemann sum approximation.

(81) $\int_4^1 2h(z) - 5dz = -\int_1^4 2h(z) - 5dz = \int_1^4 5 - 2h(z)dz = \int_1^4 5dz - 2\int_1^4 h(z)dz = 5z \Big|_1^4 - 34 = 15 - 34 = -19$.