Here are solutions to problems $2.1 : 33, 35, 48, 50, 51$ and $2.2 : 11, 12, 14$.

(33, 35) *The limit shown is $f'(a)$ for some function f and some number a. Identify f and a.*

(33)

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{\sqrt{x}-\sqrt{4}}{x-4} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

where $f(x) = \sqrt{x}$ and $a = 4$.

(35)

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = \lim_{h\to 0} \frac{\cos h - \cos 0}{h-0} = \lim_{h\to a} \frac{f(h)-f(a)}{h-a}$$

where $f(h) = \cos h$ and $a = 0$.

(48, 50, 51) *h is a function such that $h(0) = 1, h(2) = 7, h(4) = 5, h'(0) = -2, h'(2) = 3, h'(4) = -1$.*

(48) *What is the average rate of change of h over the interval $[0, 4]$?*
By definition, the average rate of change of a function f over an interval $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$. In this case we have $\frac{h(4)-h(0)}{4-0}$ which by the information given in the problem equals 1.

(50) *Evaluate* $\lim_{w\to 0} \frac{h(w)-1}{w}$.
Since $h(0) = 1$, we have that

$$\lim_{w\to 0} \frac{h(w)-1}{w} = \lim_{w\to 0} \frac{h(w)-h(0)}{w-0}$$

with the right hand side being a definition for $h'(0)$. Since we're given that $h'(0) = -2$, the limit is -2.

(51) *Evaluate* $\lim_{\Delta x\to 0} \frac{h(4+\Delta x)-5}{\Delta x}$.
Since $h(4) = 5$, we can rewrite this as:

$$\lim_{\Delta x\to 0} \frac{h(4+\Delta x)-5}{\Delta x} = \lim_{\Delta x\to 0} \frac{h(4+\Delta x)-h(4)}{\Delta x}$$

with the right hand side being a definition of $h'(4)$ (here we use the notation Δx instead of the usual h). But $h'(4) = -1$, so the value of the limit is -1.

(11) *Let* $f(w) = w^{1/2}$. *Evaluate* $f'(9)$.

By example 2 we see that $f'(w) = \frac{1}{2\sqrt{w}}$. Plugging in 9 we get $f'(9) = 1/6$.

(12) *Let* $g(t) = t^{-1}$. *Evaluate* $g'(2)$.

By example 2 we see that $g'(t) = -t^{-2}$. Thus $g'(2) = -1/4$.

(14) *Let* $g(x) = 2^x$. *Explain why Theorem 1 cannot be used to find an expression for* $g'(x)$.

Theorem 1 applies only to functions of the form $f(x) = x^k$ where k is any real constant, and $g(x) = 2^x$ is not of that form.