

Solutions to 2.4: 24, 44; 2.5: 4, 13-15, 8, 20, 26, 28, 30

- (2.4: 24) We're given that $f(x) = 1 + 2x^3 - x^4$ and we want to find the maximum and minimum values of f on the interval $[-1, 2]$. We begin by noting that $f'(x) = 6x^2 - 4x^3$. By the fact on page 112 (see example 1, p. 113), the maxima and minima of f occur only at the stationary points of f or at the endpoints of the interval, if it has any. In our case, the stationary points can be found by noting that $0 = f'(x) = 6x^2 - 4x^3 = 2x^2(3 - 2x)$, which implies that $x = 0$ or $x = 1.5$. We have that $f(0) = 1$, $f(1.5) \approx 2.68$, $f(-1) = 0$, and $f(2) = 1$. Thus f attains its largest value on $[-1, 2]$ at the point $x = 1.5$ and its smallest value at the point $x = -1$.

Note: We could have also said that since $f(x)$ is a polynomial function, and therefore continuous by a remark in the book, then since $f'(x) = 2x^2(3 - 2x)$, $f'(x)$ is positive on the interval $(-\infty, 1.5)$ and negative on the interval $(1.5, \infty)$. This means that f is increasing on $(-\infty, 1.5)$ and decreasing on $(1.5, \infty)$, which means that the maximum of f on $[-1, 2]$ will occur at $x = 1.5$, and since $f(-1) < f(2)$, the minimum occurs at $x = -1$.

- (2.4: 44) We want to find the values of x for which the slope of the tangent line to the curve $y = -x^3 + 3x^2 + 1$ takes on its largest values.

First, we calculate that $y' = -3x^2 + 6x$. This is the slope function and we want to find its largest values, NOT those of y . Since $y'(x)$ is a polynomial function and therefore differentiable, by the same fact as above (p. 112), the extrema of y' will occur at its stationary points (notice that since we're considering the interval $(-\infty, \infty)$, there are no end points to worry about). But $0 = y'' = -6x + 6$ implies that $x = 1$. Since y'' is positive and then negative around $x = 1$, $x = 1$ is a maximum of y' . Thus the maximum value of the slope of the tangent line to y occurs at $x = 1$.

Note: We were trying to find the maximum of y' , not y . Make sure that you understand why this is what the question was asking.

- (2.5: 4) A solution of the differential equation $f'(x) = 2x$ has the form $x^2 + C$, where C is any constant. Make sure that you don't forget the C !

For part *b*, there are infinitely many solutions, one for each C . For example, $x^2 + 5$, $x^2 + \pi$, etc.,

- (2.5: 8) To find the unique solution of the IVP $f'(x) = 6x + 5$, $f(0) = -3$, we first anti-differentiate f' to see that $f(x) = 3x^2 + 5x + C$, and since $f(0) = C = -3$, $C = -3$. Thus the unique solution is $f(x) = 3x^2 + 5x - 3$.

- (2.5: 13) The minus sign means that the acceleration due to gravity is downwards - towards the earth. (look on p. 126).

- (2.5: 14) In example 6 we're given the IVP: $h''(t) = -9.8$, $h(0) = 5$. If the initial velocity was 90, then we simply add another condition to our IVP that $h'(t) = 90$.

- (2.5: 15) In example 6, the cannonball is originally shot from the initial height $h(0) = 5$. If we change that to $h(0) = 3$, as the problem asks, then the resulting function h changes to: $h(t) = -4.9t^2 + 98t + 3$, and $v(t) = -9.8t + 98$. Thus the cannonball is highest at $t = 10$ (when $v(t) = 0$, the only stationary point of $h(t)$), and the height is $h(10) = 493$ meters.

To see how long the cannonball remained airborne, we want to find the roots of $h(t)$. This is because we're shooting the ball from the height of 3 meters at $t = 0$, so the value of t for which $h(t) = 0$ (it strikes the ground) will give us the total amount of time that the cannonball is airborne. Using the quadratic formula or your method of choice, we see that $h(t) = 0$ when $t = 10 + \sqrt{4930}/7 \approx 20.03$ seconds.

- (2.5: 20) We have that $y(t) = t^2/2$ and we want to see if it's a solution of the DE $y' = y$. Since $y' = t$, and $t \neq t^2/2$, $y(t)$ is not a solution to the DE.
- (2.5: 22) We have that $y(t) = .5t^4 + 1.5t^2 + .25$ and we want to check if it's a solution of the DE $y' - 2y/t = t^3$. Since $y'(t) = 2t^3 + 3t$, the left hand side of the DE is $(2t^3 + 3t) - 2(.5t^4 + 1.5t^2 + .25)/t$ which can be further simplified to $2t^3 + 3t - t^3 - 3t = t^3$. So yes, $y(t)$ is a solution to the DE.
- (2.5: 26) We have that $y = x + 1$ and we want to see if it's a solution to the DE $xy' + (y')^2 - y = 0$. Since $y' = 1$, we have that $xy' + (y')^2 - y = x + 1 - (x + 1) = 0$. Thus y is a solution to the DE.
- (2.5: 28) If $v'(2) = 1$, then at time $t = 2$ the object is accelerating at the rate of 1 meter per second per second.
- (2.5: 30) If $h''(1) = -2$, then at time $t = 1$, the object is accelerating at -2 meters per second per second.

Things to remember.

- (*) When trying to find the extrema of a differentiable function f on an interval $[a, b]$, you must find the stationary points of f , their corresponding f values, AND the values at the endpoints: $f(a)$ and $f(b)$. If you're asked about extrema of f on the interval (a, b) , then you do NOT have to test the endpoints, since they're not included. A special case of that remark is when no interval is specified (i.e., $(-\infty, \infty)$).
- (*) Understand what the FACT box on page 112 is saying.
- (*) When taking anti-derivatives, DO NOT forget the constants (" + C").
- (*) IVP simply means that you're given some values that allow you to find some or all of the constants of a differential equation.