Solutions to 2.5: 32-35, 42, 44, 39, 47; 2.6: 3, 15, 19, 31

(32) $v'(t) = -5$.

(33) $p'(t) = v(t) = 15$.

(34) $v' = kv$ and k is negative because friction works in the direction opposite to the velocity.

(35) $p'(t) = v(t) = k\sqrt{p}$.

(42) $P' = kP(3000 - P)$ where k is a constant.

(44) $O'(t) = kO(t)$

(39) Let $s(t)$ denote the position of the object at time t. Since the object's acceleration is $-8\ m/s^2$, $s''(t) = -8$. Anti-differentiating we get that the velocity of the object, $s'(t)$, is $s'(t) = -8t + C$. But since at $t = 0$ the velocity is 14, we find that $C = 14$ and $s'(t) = -8t + 14$. Anti-differentiating again we see that $s(t) = -4t^2 + 14t + D$, where D is some constant. The distance traveled by the object between time $t = 0$ and $t = 3$ is $s(3) - s(0) = -4(3)^2 + 14(3) + D - (-4(0)^2 + 14(0) + D) = 6$ meters.

(47) (a) If $P(0) = 0$, then $P' = 0$ and the size of the rabbit population remains constant (so the number of births equals the number of deaths). (b) If $P(0) = 1500$, then $P' < 0$ so the size of the rabbit population is decreasing. (c) if $P(0) = 250$, then $P' > 0$ so the size of the rabbit population is increasing. (d) The rate of change of the rabbit population is maximal at the value of P where $\frac{d}{dP}P'$ changes sign from positive to negative. But $\frac{d}{dP}P' = 2k(500 - P)$, which means that the maximal rate of change occurs at $P = 500$.

(3) If $y(t) = 2e^t$, then $y'(t) = 2e^t$ so $y(t)$ is a solution to the differential equation $y' = y$. Look at Example 1 on page 134.

(15) If $f(x) = 2e^x + \pi$, then $f'(x) = 2(e^x)' + (\pi)' = 2e^x$.

(19) If $f(x) = -2\ln(x)$, then $f'(x) = -2 * (ln(x))' = -2 * 1/x = -2/x$.

(31) To find an equation of the line tangent to $y = e^x$ at $x = 0$, we note that $y'(t) = e^x$, so the slope of the tangent line to y at $x = 0$ is $y'(0) = e^0 = 1$. The point on the y graph at $x = 0$ is $(0, 1)$, so the line is $y = x + 1$.

Here are some rules for taking derivatives that you should remember. Notice that you will often be asked problems where you have to use more than one of these to find the derivative.

(*) The derivative of any constant is always zero. One way to express this is $\frac{dC}{dx} = 0$ where C is a constant. For example, if $y = 5$, then $y' = 0$.

(*) Multiplying a function by a constant multiplies the derivative by the same constant. We can express this in a few ways: if k is a constant, then $(kf(x))' = kf'(x)$, $\frac{dkf(x)}{dx} = k\frac{df(x)}{dx}$. For example, if $f(x) = 5x^2$, then $f'(x) = 5(x^2)' = 5 * 2x = 10x$.

(*) The derivative of the sum is the sum of the derivatives. That is, if $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$. For example, if $f(x) = x^3 + e^x$, then $f'(x) = (x^3)' + (e^x)' = 3x^2 + e^x$.

(*) If you multiply the independent variable by a constant k, then the entire derivative gets multiplied by a constant. So if $f(x)$ is some function and $g(x) = f(kx)$, then $g'(x) = kf'(kx)$. For example, if $f(x) = x^2$ and $g(x) = f(5x)$, then $g'(x) = 5 * f'(5x)$, and since $f'(x) = 2x$, then $f'(5x) = 10x$, and $g'(x) = 50x$.

(*) Adding a constant to the independent variable doesn't change the derivative. That is, if $g(x) = f(x + k)$, then $g'(x) = f'(x + a)$. For example, let $f(x) = \ln(x)$ and let $g(x) = f(x+1)$. Then $g'(x) = f'(x+1) = 1/(x+1)$.

(*) Remember the power rule for taking derivatives of polynomials.

(*) Remember the derivatives of b^x and $log_b(x)$

(*) After tomorrow, know the derivatives of trig functions.

As of now, this list is quite incomplete.