

Solutions to 3.1: 10, 13, 17, 30, 49-52, 21, 22, 24, 29, 32, 53-55

- (10) Let $h(x) = x^3(x^2 - 4)$. Let $u(x) = x^3$ and $v(x) = (x^2 - 4)$. Then $h(x) = u(x)v(x)$, $u'(x) = 3x^2$, $v'(x) = 2x$, and the product rule gives $h'(x) = 3x^2(x^2 - 4) + 2x(x^3) = 3x^4 - 12x^2 + 2x^4 = 5x^4 - 12x^2$.

If we multiply first, then $h(x) = x^5 - 4x^3$ and $h'(x) = 5x^4 - 12x^2$.

- (13) Let $h(x) = e^x e^{2x}$. Let $l(x) = e^x$ and $k(x) = e^{2x}$. Then $h(x) = l(x)k(x)$, $l'(x) = e^x$, $k'(x) = 2e^{2x}$, and the product rule gives $h'(x) = e^x e^{2x} + 2e^{2x} e^x = e^{3x} + 2e^{3x} = 3e^{3x}$.

Multiplying first, $h(x) = e^{3x}$ and $h'(x) = 3e^{3x}$.

- (17) Let $h(x) = 1/e^{2x}$. Let $u(x) = 1$ and $v(x) = e^{2x}$. Then $h(x) = u(x)/v(x)$, $u'(x) = 0$, $v'(x) = 2e^{2x}$, and the quotient rule gives that $h'(x) = (0e^{2x} - 2e^{2x} \cdot 1)/(e^{2x})^2 = (-2e^{2x})/(e^{2x})^2 = -2/e^{2x}$.

Multiplying first, $h(x) = e^{-2x}$, and $h'(x) = -2e^{-2x} = -2/e^{2x}$.

- (30) Let $f(x) = \cos(x)/(x^2 + e^{2x})$. Let $u(x) = \cos(x)$ and $v(x) = x^2 + e^{2x}$. Then $u'(x) = -\sin(x)$ and $v'(x) = 2x + 2e^{2x}$. Since $f(x) = u(x)/v(x)$, the quotient rule gives $f'(x) = (-\sin(x)(x^2 + e^{2x}) - (2x + 2e^{2x})\cos(x))/(x^2 + e^{2x})^2$.

- (49) Since $g(x) = x^3 f(x)$, letting $u(x) = x^3$, $g(x) = u(x)f(x)$. $u'(x) = 3x^2$, and $f'(x)$ is "unknown", but we know that it exists. The product rule now gives that $g'(x) = 3x^2 f(x) + f'(x)x^3$. Since $f(1) = 2$ (given in the problem) and $f'(1) = 4$ (read from the graph of f'), $g'(1) = 3 \cdot 2 + 4 \cdot 1 = 10$.

- (50) We want to see if $g'(2) > 0$. Again, we have that $g'(x) = 3x^2 f(x) + f'(x)x^3$, and since $f(1) = 2$ and $f'(x) > 0$ on the interval $[1, 2]$, f is increasing on that interval and so $f(2) > 0$. Also, $f'(2) > 0$ from the graph, so $g'(2)$ is positive and g is increasing at $x = 2$.

- (51) Starting with $g'(x) = 3x^2 f(x) + f'(x)x^3$, we apply the product rule to each summand to find $g''(x)$. Letting $u(x) = 3x^2$, we have that $u'(x) = 6x$ and $(3x^2 f(x))' = 6x f(x) + 3x^2 f'(x)$. Letting $v(x) = x^3$, we have that $v'(x) = 3x^2$ and $(f'(x)x^3)' = f''(x)x^3 + 3x^2 f'(x)$. Thus $g''(x) = 6x f(x) + 3x^2 f'(x) + f''(x)x^3 + 3x^2 f'(x)$. To estimate $g''(1)$, we must find $f''(1)$. From the graph of f'' we see that $f''(1) \approx -2$, so $g''(1) \approx 6f(1) + 3f'(1) + -2 + 3f'(1) \approx 6 \cdot 2 + 3 \cdot 4 - 2 + 3 \cdot 4 = 34$.

- (52) From the above, $g''(x) = 6x f(x) + 6x^2 f'(x) + x^3 f''(x)$, so $g''(-2) = -12f(-2) + 24f'(-2) - 8f''(-2)$. We notice that $f(-2) \leq -1$ since $f(1) = 2$, $f'(x) \geq 1$ if $-2 \leq x \leq 1$ (speed limit law), $f'(-2) = 1$, and $f''(-2) \approx 4$. Thus, $g''(-2) = -12(-1) + 24 - 8 \cdot 4 = 4 > 0$, so g is concave up at $x = -2$.

- (21) Let $f(x) = x \sin(x)$. Letting $r(x) = x$ and $s(x) = \sin(x)$, $r'(x) = 1$ and $s'(x) = \cos(x)$. Then the product rule gives that $f'(x) = (r(x)s(x))' = 1 \sin(x) + x \cos(x)$.

- (22) Let $f(x) = x^2 \cos(x)$. Letting $u(x) = x^2$ and $l(x) = \cos(x)$, $f(x) = u(x)l(x)$, $u'(x) = 2x$, $l'(x) = -\sin(x)$, and $f'(x) = 2x \cos(x) - \sin(x)x^2$ by the product rule.
- (24) Let $f(x) = x^3 e^x$. Letting $u(x) = x^3$ and $v(x) = e^x$, $u'(x) = 3x^2$, $v'(x) = e^x$, and the product rule gives that $f'(x) = 3x^2 e^x + x^3 e^x$.
- (29) Let $f(x) = \sin(x)/x^2$. Letting $u(x) = \sin(x)$ and $v(x) = x^2$, $u'(x) = \cos(x)$, $v'(x) = 2x$, and the quotient rule gives that $f'(x) = (\cos(x)x^2 - \sin(x)2x)/(x^2)^2$.
- (32) Let $f(x) = \sin(x)/\cos(2x)$. Then if $u(x) = \sin(x)$ and $v(x) = \cos(2x)$, $u'(x) = \cos(x)$, $v'(x) = -2\sin(2x)$, and the quotient rule gives $f'(x) = (\cos(x)\cos(2x) + 2\sin(x)\sin(2x))/\cos^2(2x)$.
- (53) $g(x) = \sin(x)f(x)$. By the product rule, $g'(x) = \cos(x)f(x) + \sin(x)f'(x)$. Thus $g'(0) = \cos(0)f(0) + \sin(0)f'(0) = f(0) = 3$.
- (54) We want to know if $g'(-\pi/2) > 0$. From the above, $g'(-\pi/2) = \cos(-\pi/2)f(-\pi/2) + \sin(-\pi/2)f'(-\pi/2) = -f'(-\pi/2) > 0$ (from the graph of f'). So yes, g is increasing at $-\pi/2$.
- (55) We want to know if $g'(\pi/2) > 0$. Using the same idea as above, $g'(\pi/2) = \cos(\pi/2)f(\pi/2) + \sin(\pi/2)f'(\pi/2) = f'(\pi/2) > 0$ from the graph. Thus g is increasing at $\pi/2$.