Solutions to 3.2: 9, 11, 12, 18, 19, 28, 37. 3.3: 1. Homework Problems

(9) Let $f(x) = \sin(e^{3x})$. Then $f(x) = u(v(q(x)))$ where $u(x) = \sin(x)$, $v(x) = e^x$ and $q(x) = 3x$. We now have that $u'(x) = \cos(x)$, $v'(x) = e^x$ and $q'(x) = 3$, and by the chain rule $f'(x) = u'(v(q(x)))v'(q(x))q'(x) = \cos(e^{3x})e^{3x}3$.

(11) Let $f(x) = \ln(4x^2 + 3)$. Then $f(x) = \ln(u(x))$ where $u(x) = 4x^2 + 3$. Since $(\ln(x))' = 1/x$ and $u'(x) = 8x$, the chain rule gives $f'(x) = \ln'(u(x))u'(x) = \frac{1}{4x^2+3}8x$.

(12) Let $f(x) = 1/\sqrt{x^2 + e^{-3x}}$. Letting $u(x) = 1/x$, $v(x) = \sqrt(x)$, $q(x) = x^2 + e^{-3x}$ we have that $f(x) = u(v(q(x)))$. Now, $u'(x) = -1/x^2$, $v'(x) = 1/(2\sqrt{x})$, and $q'(x) = 2x - 3e^{-3x}$. Using the chain rule we get that $f'(x) = \frac{-1}{x^2+e^{-3x}} \frac{1}{2\sqrt{x^2+e^{-3x}}}(2x - 3e^{-3x})$.

(18) Let $f(x) = (1 - 2x^4)^{1/3}$. Let $u(x) = (1 - 2x^4)$. Then $u'(x) = -8x^3$, and $f(x) = (u(x))^{1/3}$. The chain rule now gives $f'(x) = 1/3(u(x))^{-2/3}u'(x) = 1/3(1 - 2x^4)^{-2/3}(-8x^3)$.

(19) Let $f(x) = \ln(x^2 + 3\sin(x))$. Let $u(x) = x^2 + 3\sin(x)$. Then $u'(x) = 2x + 3\cos(x)$, $f(x) = \ln(u(x))$, and the chain rule gives $f'(x) = 1/(u(x))u'(x) = (2x + 3\cos(x))/(x^2 + 3\sin(x))$.

(28) If $k(x) = f(x^3)$, then $k'(x) = f'(x^3)(3x^2)$ by the chain rule. $k'(-1) = f'((-1)^3)(3(-1)^2) = 3f'(-1) = 3 * 4 = 12$.

(37) Let $F(x) = 2x^2e^{x^2} - 4$. Then $F'(x) = 2(2xe^{x^2} + x^2(e^{x^2}2x)) = (4x + 4x^3)e^{x^2} \neq f(x)$. Thus the answer is "no."

(1) We know from example 4 that $\frac{dy}{dx} = \frac{6y-10x}{10y-6x}$. So the derivative is 0 if and only if $6y - 10x = 0$, or equivalently $y = 5x/3$. Substituting that into the equation of the ellipse and solving for x and then y we see that the points are $(3/\sqrt{5}, \sqrt{5})$ and $(-3/\sqrt{5}, -\sqrt{5})$.

Homework Problems due Thursday

(1) Use the limit definition of the derivative to find the derivative of each of the following functions:

  (a) $f(x) = 1/x$
  (b) $f(x) = x^2 - 5$
  (c) $f(x) = 5\sqrt{x}$.

(2) For each function below, find the equation of the line tangent to the function at $x = 1$.

  (a) $f(x) = 3x^2 - 5\ln(x) + 1$

(b) $g(x) = \sqrt{x} - 3\cos(x\pi) + 3$

(3) Appendix E, problems 29-45.

(4) Section 3.5, exercises 3, 6, 14, 15, 16, 25, 26, 33, 40, 53.