Solutions to: 3.3: 4, 15b, 17b, 16b, 18b, 20, 21, 8 Appendix E Section 3.5 exercises

(4) We're given that $4x^2y - 3y = x^3$. Taking $\frac{d}{dx}$ (the derivative with respect to x of both sides) we get $8xy + 4x^2y' - 3y' = 3x^2$. Solving for y' gives $y' = \frac{3x^2-8xy}{4x^2-3}$.

We can also solve for y explicitly from the given equation: $y(4x^2 - 3) = x^3$, which means that $y = \frac{x^3}{4x^2-3}$. Using the quotient rule we get that $y' = \frac{4x^4-9x^2}{(4x^2-3)^2}$.

(15b) Our equation is $x^2 + 2y^2 = 4$. Taking the derivative with respect to x of both sides we get: $2x + 4yy' = 0$. Solving for y' gives $y' = \frac{-x}{2y}$.

(17b) We're given $y^2 - x^2 = x^2y^2$. Differentiating both sides with respect to x gives: $2yy' - 2x = 2xy^2 + 2x^2yy'$. Solving for y' we get $y' = \frac{x+xy^2}{y-x^2y}$.

(16b) We begin with $4x^2 - 9y^2 = 36$. Taking the derivative with respect to x of both sides we get: $8x - 18yy' = 0$. Solving for y' gives $y' = \frac{4x}{9y}$.

(18b) Given $y^2(2 - x) = x^3$, first multiply out to get: $2y^2 - xy^2 = x^3$. Now take $\frac{d}{dx}$ of both sides: $4yy' - (y^2 + 2xyy') = 3x^2$. Solving for y' we get $y' = \frac{3x^2-y^2}{4y-2xy}$.

(20) Given the equation $x^{2/3}+y^{2/3} = 2$, we first need to find $\frac{dy}{dx}$. Differentiating both sides with respect to x we get $(2/3)x^{-1/3} + (2/3)y^{-1/3}y' = 0$. Thus $y' = \frac{-y^{1/3}}{x^{1/3}}$. At the point $(1, 1)$, $y' = -1$. Using the point slope formula we get that $y - 1 = -1(x - 1)$, or equivalently $y = -x + 2$.

(21) We start with $2(x^2 + y^2)^2 = 25xy$. Differentiating both sides with respect to x gives: $4(x^2 + y^2)(2x + 2yy') = 25xy' + 25y$. So $y' = \frac{25y-8x(x^2+y^2)}{8y(x^2+y^2)-25x}$. Thus at the point $(1, 2)$, $y' = 2/11$ and the equation of the tangent line is $y = (2x + 20)/11$.

(8) We first implicitly differentiate $x^5 + xy^3 + x^2y + y^5 = 32$ to get $5x^4 + (y^3 + 3xy^2y') + (2xy + x^2y') + 5y^4y' = 0$. Solving for y' we get $y' = \frac{5x^4+y^3+2xy}{3xy^2+x^2+5y^4}$. At the point $(0, 2)$ the slope of the tangent line is $-1/10$.

Appendix E: 29-45

(29) $8^{2/3} = 4$ translates to $\log_8(4) = 2/3$.

(30) $10^3 = 1000$ translates to $\log_{10}(1000) = 3$.

(31) $10^{-4} = 0.0001$ translates to $\log_{10}(0.0001) = -4$.

(32) $(1/2)^2 = 4$ translates to $\log_{1/2}(4) = 2$.

(33) $\log_2(7) = x$ translates to $2^x = 7$.

(34) $\log_4(4) = 1$ translates to $4^1 = 4$.

(35) $\log_5(1/25) = -2$ translates to $5^{-2} = 1/25$.

(36) $\log_{10}(1000) = 3$ translates to $10^3 = 1000$.

(37) $\log_{64}(128) = 7/8$ translates to $64^{7/8} = 128$.

(38) $\log_{1/2}(1/4) = 2$ translates to $(1/2)^2 = 1/4$.

(39) $2^{\log_2(16)} = 16$.

(40) $2^{\log_2(1/2)} = 1/2$.

(41) $10^{\log_{10}(7)} = 7$.

(42) $\log_6(\sqrt{6}) = \log_6(6^{1/2}) = 1/2$.

(43) $\log_5(5^3) = 3\log_5(5) = 3$.

(44) $\log_3(1/27) = \log_3(3^{-3}) = -3$.

(45) $e^{2\ln(x)} = (e^{\ln(x)})^2 = x^2$.

Section 3.5

(3) $r(\theta) = \tan(3\theta)$. Recall that $\tan'(\theta) = \sec^2(\theta)$. By the chain rule, $r'(\theta) = 3\sec^2(3\theta)$.

(6) $h(z) = \ln(\pi\sin(e^z))$. Applying the chain rule we see that $h'(z) = \frac{1}{\pi\sin(e^z)}\pi\cos(e^z)e^z = e^z\cot(e^z)$.

(14) $A(z) = z^{-4}\tan(3z)$. Using the product rule, and the chain rule we get that $A'(z) = -4z^{-5}\tan(3z) + z^{-4}\sec^2(3z)3$.

(15) $f(x) = x\sin(x)$. Using the product rule we get that $f'(x) = x\cos(x) + \sin(x)$.

(16) $g(t) = \sqrt{1 + t^3\cos(t)}$. It's easiest to rewrite this expression as $g(t)\ (1 + t^3\cos(t))^{1/2}$, use the chain rule, and then the product rule for $t^3\cos(t)$. Altogether we get $g'(t) = \frac{1}{2\sqrt{1+t^3\cos(t)}}(3t^2\cos(t) - t^3\sin(t))$.

(25) $f(x) = (e^{2x} - 3\ln(x))^4$. Using the chain rule, $f'(x) = 4(e^2x - 3\ln(x))^3(2e^2x - 3/x)$.

(26) $f(x) = \frac{x}{1+x^2}$. Using the quotient rule, $f'(x) = \frac{1-x^2}{(1+x^2)^2}$

(33) $f(x) = \frac{\sin(x+1)}{e^x+e^{-x}}$. $f'(x) = \frac{\cos(x+1)(e^x+e^{-x})-\sin(x+1)(e^x-e^{-x})}{(e^x+e^{-x})^2}$.

(53) $f(x) = \sin^2(x\cos(x))$. Use the chain rule combined with the product rule to get that $f'(x) = 2\sin(x\cos(x))\cos(x\cos(x))(\cos(x) - x\sin(x))$.