Math 220(BE1) Exam 4

Name:	
Problem 1:	/21
Problem 2:	/5
Problem 3:	/25
Problem 4:	/10
Problem 5:	/9
Problem 6:	/15
Problem 7:	/15
Total:	/100

No calculators, books, cell phones, or notes are to be used during the test. This exam covers material in chapter 5. You must show your work to recieve credit

1) (7 points each) Evaluate the following integrals.

(a) $\int_0^{\pi} \cos(x)+5e^{3x}\, dx$

An antiderivative of $\cos(x) = \sin(x)$ and an antiderivative of $5e^{3x} = \frac{5}{3}e^{3x}$, so the integral is $\sin(\pi) + \frac{5}{3}e^{3\pi} - (\sin(0) + \frac{5}{3}e^0 = \frac{5}{3}(e^{3\pi} - 1)$.

(b) $\int_1^e \frac{dt}{t(2 + \ln t)}$

Let $u = 2+\ln(t)$, then $du = \frac{1}{t}dt$ and $u(1) = 2+\ln(1) = 2$, $u(e) = 2+\ln(e) = 3$. The integral then becomes $\int\limits_2^3 \frac{1}{t}dt = \ln(|u|)|_2^3 = \ln(3) - \ln(2)$.

(c) $\int \frac{\cos(x)\sin^3(x)}{1 + \sin^2(x)}\, dx$

Let $u = 1 + \sin^2(x)$. Then $du = 2\sin(x)\cos(x)$, and the integral becomes $\frac{1}{2}\int \frac{u-1}{u}du$, which antidifferentiates to $\frac{1}{2}u - \frac{1}{2}\ln(|u|) + C$. Plugging x back in we get $\frac{1}{2}(1 + \sin^2(x)) + \frac{1}{2}\ln(1 + \sin^2(x)) + C$.

2) (5 points) State either formal version of the Fundamental Theorem of Calculus. Either form from the TEXT BOOK would have been acceptable.

3) (5 points each) Let $F(x) = \int_1^x f(t)\, dt$. The graph of $f(x)$ is given below. Answer the following questions about F. Justify your answers.

(a) Where is $F(x)$ increasing? $F(x)$ is increasing where $f(x) > 0$. That is, $(-\infty, -6) \cup (-3, 2) \cup (7, \infty)$.

(b) Where is $F(x)$ concave up? $F(x)$ is concave up where $f(x)$ is increasing. These intervals are $(-5, 0) \cup (4, 5) \cup (6, \infty)$.

(c) Where does $F(x)$ have stationary points? $F(x)$ has stationary points where $f(x) = 0$. These points are $x = -6, -3, 2, 5, 7$.

(d) Where does $F(x)$ have a local minimum? By the first derivative test, $F(x)$ has a local minimum at $x = -3, 7$.

(e) Suppose $G(1) = -4$ and $G'(x) = f(x)$. How are the graphs of F and G related? Be specific! Since $F(x) = G(x) + C$, we get that $F(1) = G(1) + C$. Since $F(1) = 0$ and $G(1) = -4$, $C = 4$. Thus $F(x) = G(x) + 4$.

4) (5 points each) The graph of $f(x)$ is given below.

(a) Approximate $\int_2^5 f(x)\,dx$ by a *left* sum with three equal subintervals. Does this sum over-estimate the actual value of the integral? The areas of the rectangles are $6 + 3 + 2 = 11$. This is an overestimate of the value of the integral.

(b) Find upper and lower bounds for $\int_0^6 f(t)\,dt$. The minimum of f on $[0, 6]$ is 1, and the maximum is 7. Thus the lower bound is $6(1) = 6$, and the upper bound is $6(7) = 42$.

5) (9 points) Let $f(x) = 2e^x$. Write $\int_0^4 f(x)\,dx$ as a limit of *right* Riemann sums. Express your answer in sigma notation. The width is $\frac{4}{n}$, and $x_i = \frac{4i}{n}$. The integral is then written as

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 2e^{\frac{4i}{n}}$$

6) (15 points) Evaluate the following limit. Use the interval $[1, 4]$.

$$\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(1 + \frac{3i}{n}\right)^{-1/2}$$

Since the width is $\frac{3}{n}$, and $x_i = 1 + \frac{3i}{n}$, we see that this sum is a limit definition of $\int_1^4 \frac{1}{\sqrt{x}} dx = 2$.

7) (15 points) Sketch the region bounded above by $y = 3x$ and below by $y = x^2 - 4$. Find its area.

$3x = x^2 - 4$ imply that $x = -1, 4$, which are the x coordinates of the intersection of the two curves. As $3x$ is always bigger than $x^2 - 4$ on that interval, the area between the curves is given by $\int_1^4 3x - (x^2 - 4)dx$. Evaluating that is easy.