

Math 220 BE1 Practice Exam 3

No calculators, books, cell phones, or notes are to be used during the test.

This exam covers sections 4.2 through 4.9. You must show your work to receive credit. Answer **all** questions.

1) State in full

- (a) The Mean Value Theorem
- (b) The Extreme Value Theorem
- (c) Rolle's Theorem
- (d) The Intermediate Value Theorem

2) Find the following limits

- (a)  $\lim_{x \rightarrow \infty} e^{-x} \ln(x)$
- (b)  $\lim_{x \rightarrow 0} \frac{1}{\cos(x)}$
- (c)  $\lim_{x \rightarrow \pi} \frac{\cos(x-\pi)-1}{(x-\pi)^3}$ .

3) (a) A triangle has legs on the positive  $x$ -axis and  $y$ -axes, and its hypotenuse passes through the point  $(2, 1)$ . Which such triangle has the smallest area?

\* (b) Find all the critical points of  $|x^2 - x|$ . *Hint: Rewrite the function as a piece-wise function to remove the absolute value.*

4) Graph the parametric curve given by  $x = 2 \cos(t)$ ,  $y = t$ ,  $-\pi \leq t \leq \pi$ . Label both axes and indicate the points corresponding to  $t = 0$  and  $t = -\pi$  on the graph.

5) A lamp positioned on top of a 12 foot pole casts a shadow on a 6 foot man who is walking away from the lamp at the rate of 1 foot per second. How fast is the length of the shadow changing when the man is 8 feet away from the lamp?

6) (a) Find a third-order Taylor polynomial for  $f(x) = e^x$  expanded around  $x_0 = 0$ .

(b) Use the polynomial from part a to estimate the value of  $e$ .

7) Let  $f(x) = |x - 1|$ .

(a) Does  $f$  satisfy the hypothesis of the Initial Value Theorem on the interval  $[-1, 1]$ ? Justify your answer.

(b) Does  $f$  satisfy the hypothesis of the Initial Value Theorem on the interval  $[-1, 1]$ ? Justify your answer.

(c) Does  $f$  satisfy the *conclusion* of the Mean Value Theorem on the interval  $[-1, 1]$ ? Justify your answer.