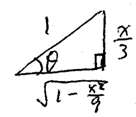
Solutions to Practice Problems

1. $\int \sin^4 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1-2\cos x+\cos^2 2x)\,dx$

$= \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{4}\int \frac{1}{2}(1+\cos 4x)\,dx = \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

2. Let $x = 3\sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, then $dx = 3\cos\theta\, d\theta$

$\int \frac{dx}{(\sqrt{9-x^2})^3} = \int \frac{3\cos\theta\, d\theta}{(3\cos\theta)^3} = \frac{1}{9}\int \sec^2\theta\, d\theta$



$= \frac{1}{9}\tan\theta + C = \frac{1}{9}\left(\frac{x/3}{\sqrt{1-\frac{x^2}{9}}}\right) + C = \frac{1}{9}\left(\frac{x}{\sqrt{9-x^2}}\right) + C$

3. $\int \sin^{-1}x\,dx = x\sin^{-1}x - \int x\,d\sin^{-1}x = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$

but $\int \frac{x}{\sqrt{1-x^2}}dx = \int \frac{\frac{1}{(-2)}du}{\sqrt{u}} = -\sqrt{u} + C = -\sqrt{1-x^2} + C$ $\left(\begin{array}{l}\text{let } u = 1-x^2 \\ du = -2x\,dx\end{array}\right)$

$\therefore \int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2} + C$

4. $\int \frac{x^4}{x^2+4}dx = \int \left(\frac{x^4-16}{x^2+4} + \frac{16}{x^2+4}\right)dx = \int \left(x^2 - 4 + \frac{16}{x^2+4}\right)dx$

$= \frac{1}{3}x^3 - 4x + 16\int \frac{dx}{x^2+4} = \frac{1}{3}x^3 - 4x + 16\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

$= \frac{1}{3}x^3 - 4x + 8\tan^{-1}\frac{x}{2} + C$

5. $\int x\cos x\,dx = x\sin x - \int \sin x\,dx$ $\left(\begin{array}{ll} u = x & v = \sin x \\ du = dx & dv = \cos x\,dx\end{array}\right)$

$= x\sin x + \cos x + C$

6. $\int_0^1 \frac{1}{\sqrt{x}}dx = \lim_{t\to 0}\int_t^1 \frac{1}{\sqrt{x}}dx = \lim_{t\to 0}\left(2\sqrt{x}\Big|_t^1\right) = \lim_{t\to 0}(2 - 2\sqrt{t}) = 2$

7. $\int_1^\infty \frac{1}{\sqrt{x}}dx$ is divergent. In fact, (see over)

$$\int_1^\infty \frac{dx}{\sqrt{x}} = \lim_{t\to\infty} \int_1^t \frac{dx}{\sqrt{x}} = \lim_{t\to\infty} \left(2\sqrt{x}\Big|_1^t\right) = \lim_{t\to\infty} (2\sqrt{t} - 2) = \infty .$$

8. Now $\tan\theta = x$. $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then $x=0$, i.e. $\tan\theta = 0$ $\Rightarrow \theta = 0$. and $x=1$ i.e. $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$.
$\therefore a = 0$, $b = \frac{\pi}{4}$ in $\int_a^b \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^2}$

9. $$\frac{1}{x^2(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$$

10. Right endpoint & Midpoint will overestimate the integral.

11. Omit (Hint: $\ln 5 = \int_1^5 \frac{dx}{x}$)

12. Now $S_{2n} = 0.7$, $T_n = 0.5$ but $\frac{1}{3}T_n + \frac{2}{3}M_n = S_{2n}$
$\therefore M_n = \frac{3}{2}(S_{2n} - \frac{1}{3}T_n) = \frac{1}{2}(3S_{2n} - T_n) = \frac{1}{2}(3\cdot 0.7 - 0.5) = 0.8$

13. $f(x) = x^4$, $f''(x) = 12x^2 \le 12\cdot 2^2 = 48$ for $0 \le x \le 2$
$\therefore |E_{T_4}| \le \frac{48(2-0)^3}{12\cdot 4^2} = 1$