Problem 1

$$\int \sin^4 x dx \quad =$$

Problem 2

$$\int \frac{1}{(\sqrt{9-x^2})^3} dx \quad =$$

Problem 3

$$\int \sin^{-1} x dx \quad =$$

Problem 4

$$\int \frac{x^4}{x^2+4} dx \quad =$$

Problem 5

$$\int x \cos x dx \quad =$$

Problem 6

$$\int_0^1 \frac{1}{\sqrt{x}} dx \quad =$$

Problem 7

$$\int_1^\infty \frac{1}{\sqrt{x}} dx \quad =$$

Problem 8 Consider the integral

$$\int_0^1 \frac{1}{(1+x^2)^2} dx$$

Make the substitution $x = \tan\theta$, then $dx = \sec^2\theta$. What will the new limits of integration be? i.e., what is $a =?$, $b =?$ in

$$\int_a^b \frac{\sec^2\theta}{(1+\tan^2\theta)^2} d\theta$$

Problem 9 Write $\frac{1}{x^2(x^2+1)^2}$ as a sum of partial fractions. You need not determine the coefficients A, B, C etc.

Problem 10 Suppose $y = f(x)$ on the interval $[a, b]$. Suppose that $y' > 0$ on $[a, b]$, and $y'' < 0$ on $[a, b]$. Wchich of the following will overestimate the integral $\int_a^b f(x)dx$?

left endpoint right endpoint midpoint trapezoidal

Problem 11 Use each rule above plus Simpson's rule to estimate $\ln 5$. Use $n = 4$

Problem 12 Consider $\int_a^b f(x)dx = A$. Suppose Simpson's rule gives an approximation of 0.7 for A and the trapeziodal rule gives the estimate 0.5. What would the mid point rule estimate be?

Problem 13 Suppose the definite integral

$$\int_0^2 x^4 dx$$

is approximated using the trapeziodal rule with n=4. Find a bound on the error $|E_{T_4}| \leq$? Recall that

$$|E_{T_4}| \leq \frac{K(b-a)^2}{12n^2}$$