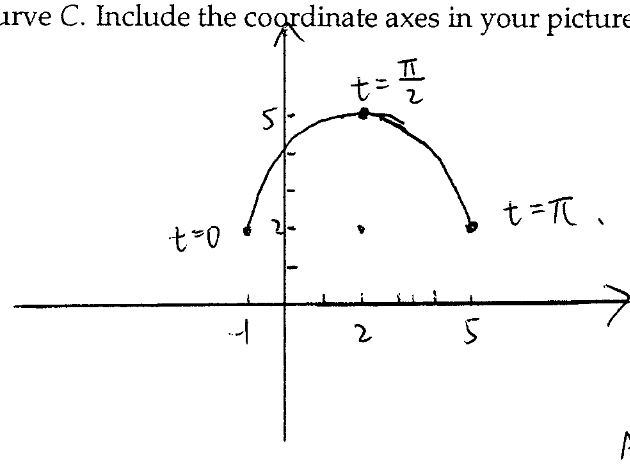
Math 241 (10am): Quiz 3 Your Name Here: Solution

1. Consider the curve C in $\mathbb{R}^2$ given by the parameterization $\mathbf{r}\colon [0,\pi] \to \mathbb{R}^2$ where

$$\mathbf{r}(t) = (2 - 3\cos t, 2 + 3\sin t)$$

(a) Sketch the curve C. Include the coordinate axes in your picture for scale. (2 points)

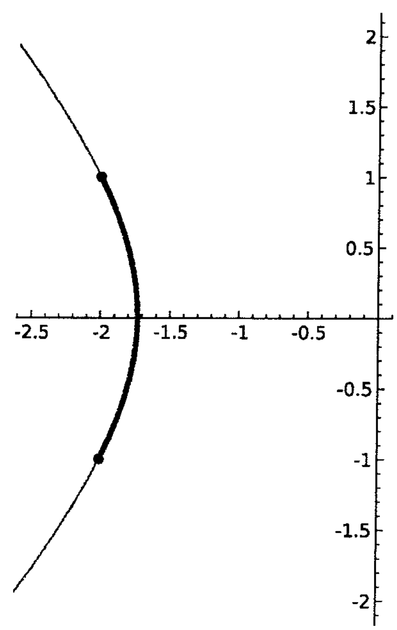


$x = 2 - 3\cos t$
$y = 2 + 3\sin t$.
then $(x-2)^2 + (y-2)^2 = 9$.
and. since t is from 0 to π, it is. a upper semicircle.
And you can see this if you look at some special points.
For example : $t = 0, \frac{\pi}{2}, \pi$.

(b) For $f(x,y) = x^3 + y$, reduce the line integral $\int_C f\,ds$ to an ordinary definite integral (something like $\int_0^2 t^2 \sin t\,dt$), but do not evaluate it. (2 points)

$$\int_C f\,ds$$
$$= \int_0^\pi f(r(t)) \cdot |r'(t)|\,dt$$
$$= \int_0^\pi \left((2-3\cos t)^3 + 2+3\sin t\right) \cdot \sqrt{(3\sin t)^2 + (3\cos t)^2}\,dt$$
$$= 3\int_0^\pi \left((2-3\cos t)^3 + 2+3\sin t\right) dt$$

2. Consider the curve C which is the darker portion of the hyperbola $x^2 - y^2 = 3$ between the two marked points. Give a parameterization $\mathbf{r}$ of C, indicating the domain so that it traces out precisely the segment indicated. (3 points)



$x^2 - y^2 = 3$.
y is from -1 to 1.
x is negative.
$\Rightarrow$ $x = -\sqrt{3+y^2}$ and $y \in [-1, 1]$
so $r(t) = (-\sqrt{3+t^2}, t)$
Domain : $[-1, 1]$