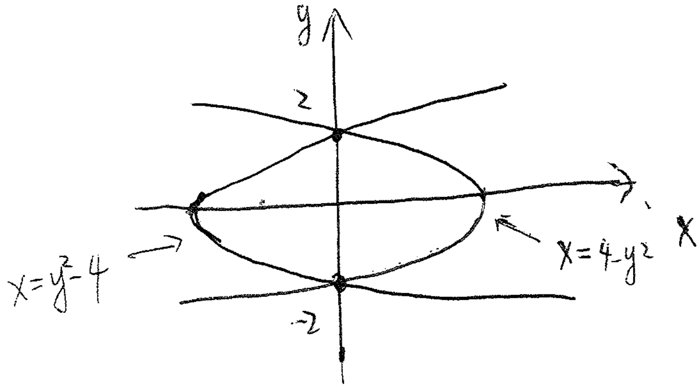
Math 241 (10am): Quiz 4 Your Name Here: Solution.

1. Consider the region R in $\mathbb{R}^2$ bounded by the parabolas $y^2 - x - 4 = 0$ and $y^2 + x - 4 = 0$

   (a) Sketch the region R. Label the points where two parabolas meet. (2 points)



   (b) Write down a double integral which computes the area of R and evaluate it. (4 points)

$$\int_{-2}^{2}\int_{y^2-4}^{4-y^2} dx\,dy. \qquad \{R = (x,y) \mid -2 \le y \le 2,\ y^2-4 \le x \le 4-y^2\}$$

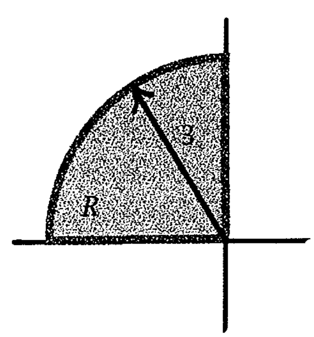
$$= \int_{-2}^{2} 8-2y^2\,dy$$

$$= 8y - \frac{2}{3}y^3 \Big|_{-2}^{2}$$

$$= 16 - \frac{16}{3} - \left(-16 + \frac{16}{3}\right)$$

$$= \frac{64}{3}$$

2. Let R be the region shown at right. Use polar coordinates to evaluate $\iint_R x\,dA$. (4 points)



$$\int_{\frac{\pi}{2}}^{\pi}\int_0^3 r\cos\theta \cdot r\,dr\,d\theta \qquad R = \begin{array}{l} 0 \le r \le 3. \\ \frac{\pi}{2} \le \theta \le \pi. \end{array}$$

$$= \int_{\frac{\pi}{2}}^{\pi}\int_0^3 r^2\cos\theta\,dr\,d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi} \frac{r^3}{3}\cos\theta \Big|_{r=0}^{3} d\theta.$$

$$= \int_{\frac{\pi}{2}}^{\pi} 9\cos\theta\,d\theta$$

$$= 9\sin\theta \Big|_{\frac{\pi}{2}}^{\pi}$$

$$= 9\sin\pi - 9\sin\frac{\pi}{2} = -9.$$