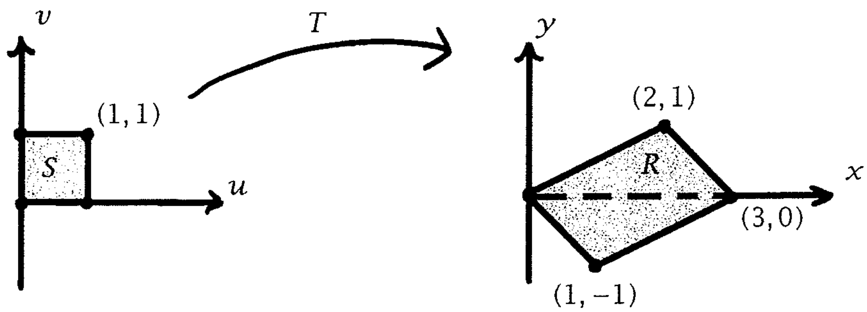
Math 241 (10am): Quiz 5 Your Name Here: Solution

Consider the region R in $\mathbb{R}^2$ shown at right.



1. Find a transformation T from the unit square $S = [0,1] \times [0,1]$ to R. (3 points)

$T(u,v) = (au + bv,\ cu + dv)$.

$T(0,1) = (2,1) \Rightarrow b = 2,\ d = 1$.

$T(1,0) = (1,-1) \Rightarrow a = 1,\ c = -1$

So $T(u,v) = (u + 2v,\ -u + v)$

or $x = u + 2v,\quad y = -u + v$.

2. Use your answer in (a) to evaluate the following via an integral over S. (4 points)

$$\iint_R xy\, dA$$

$\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 3$.

$$\iint_R xy\, dA$$

$$= \int_0^1 \int_0^1 (u+2v)(-u+v) \cdot 3\, du\, dv$$

$$= 3 \int_0^1 \int_0^1 (-u^2 - uv + 2v^2)\, du\, dv.$$

$$= 3 \int_0^1 \left(-\frac{1}{3}u^3 - \frac{1}{2}u^2 v + 2uv^2\right)\Big|_{u=0}^{1} dv$$

$$= 3 \int_0^1 \left(-\frac{1}{3} - \frac{1}{2}v + 2v^2\right) dv.$$

$$= 3\left(-\frac{1}{3}v - \frac{1}{4}v^2 + \frac{2}{3}v^3\right)\Big|_0^1$$

$$= 3\left(-\frac{1}{3} - \frac{1}{4} + \frac{2}{3}\right).$$

$$= -1 - \frac{3}{4} + 2$$

$$= \frac{1}{4}$$