Solution of Work Sheet on October 26th

1. a). $\int_{-1}^{1}\int_{0}^{1}(6-2x-3y)\,dy\,dx.$

$= \int_{-1}^{1}\left(6y-2xy-\frac{3}{2}y^2\right)\Big|_0^1\,dx$

$= \int_{-1}^{1}\left(6-2x-\frac{3}{2}\right)dx$

$= \frac{9}{2}x-x^2\Big|_{-1}^{1}$

$= \frac{9}{2}-1-\left(\frac{9}{2}\cdot(-1)-1\right)$

$= 9$

b). $\int_0^1\int_{-1}^{1}(6-2x-3y)\,dx\,dy$

$= \int_0^1 (6x-x^2-3xy)\Big|_{-1}^{1}\,dy$

$= \int_0^1 6-1-3y-(-6-1+3y)\,dy$

$= \int_0^1 (12-6y)\,dy$

$= 12y-3y^2\Big|_0^1$

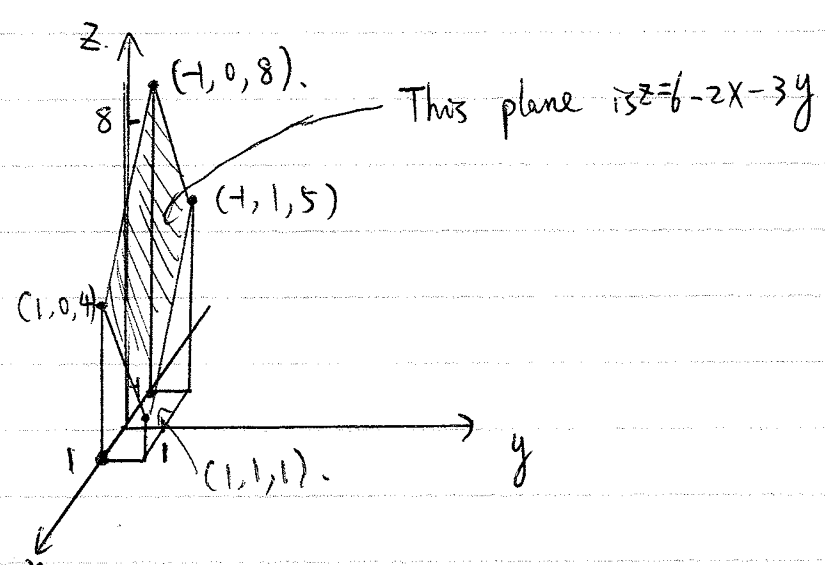
$= 9.$

c). When $-1 \le x \le 1$, $0 \le y \le 1$, $6-2x-3y$ got its minimum when $x=1$, $y=1$, and $6-2-3=1>0$.

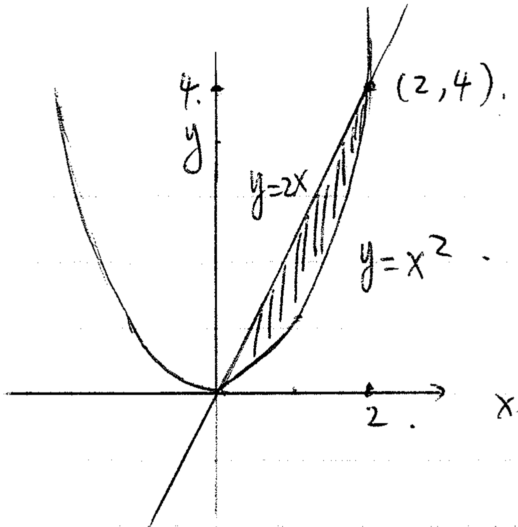
Why $(1,1)$ is minimum? Because when x or y increase, $6-2x-3y$ decrease!

And since $6-2x-3y$ is positive over the region $R=[-1,1]\times[0,1]$, the integral should also be non-negative.

d).



2. a).



$y = 2x$ $\quad 2x = x^2$

$y = x^2 \quad \Rightarrow x = 0$, or 2.

(Check Example 2 on Page 968).

b). $\int_0^2 (2x - x^2)\, dx$.

c). $\int_0^2 \int_{x^2}^{2x} dy\, dx$

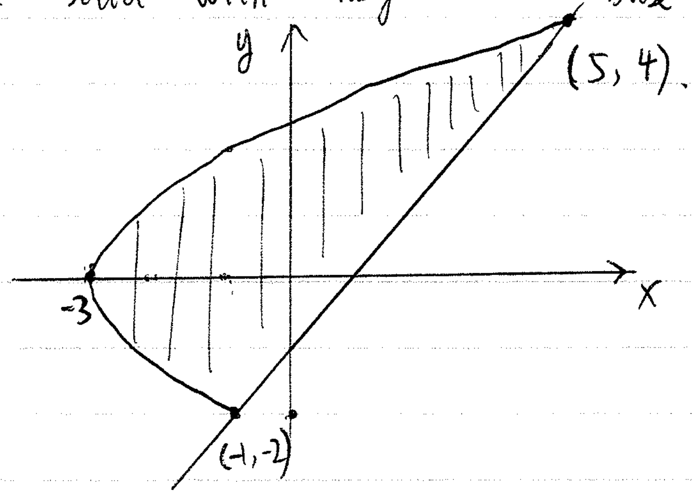
$= \int_0^2 (2x - x^2)\, dx$

$= x^2 - \frac{1}{3}x^3 \Big|_0^2$

$= 4 - \frac{8}{3} = \frac{4}{3}$

This double integral is $\int_0^2 \int_{x^2}^{2x} 1\, dy\, dx$, so it calculates a solid with height 1 and base be the region R.

3.



(Check Example 3 on Page 968)

b). $-2x + y^2 = 6 \Rightarrow x = \frac{y^2}{2} - 3$.

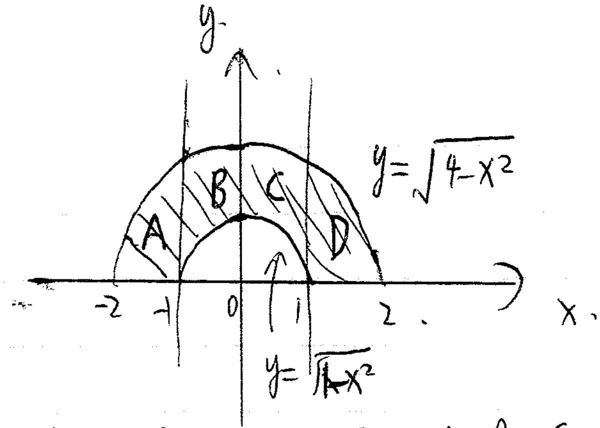
$-x + y = -1 \Rightarrow x = y + 1$

And the region $R = \{(x,y) \mid -2 \leq y \leq 4,\ \frac{1}{2}y^2 - 3 \leq x \leq y+1\}$.

$\int_{-2}^{4} \int_{\frac{1}{2}y^2-3}^{y+1} dx\,dy = \int_{-2}^{4} (y+1-(\frac{1}{2}y^2-3))\, dy = 18$.

c). $\dfrac{\int_{-2}^{4} \int_{\frac{1}{2}y^2-3}^{y+1} (x+y)\, dx\, dy}{\int_{-2}^{4} \int_{\frac{1}{2}y^2-3}^{y+1} dx\,dy} = \dfrac{6}{5}$.

4.



Here we have four regions A, B, C, D, but since.

Area of A = Area of D and Area of B = Area of C.

We only need to caculate C and D.

$C \Rightarrow. \int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} dy\, dx.$

$D \Rightarrow. \int_1^2 \int_0^{\sqrt{4-x^2}} dy\, dx.$

So totally we have $2\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} dy\, dx + 2\int_1^2 \int_0^{\sqrt{4-x^2}} dy\, dx$

$$= 2\int_0^1 \sqrt{4-x^2}\,dx - 2\int_0^1 \sqrt{1-x^2}\, dx + 2\int_1^2 \sqrt{4-x^2}\, dx$$

$$= 2\int_0^2 \sqrt{4-x^2}\, dx - 2\int_0^1 \sqrt{1-x^2}\, dx.$$

And $2\int_0^2 \sqrt{4-x^2}\, dx.$

$$\overset{x=2\sin\theta}{=\!=} 2\int_0^{\frac{\pi}{2}} 2\cos\theta\, d(2\sin\theta).$$

$$= 8\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta.$$

$$= 8\cdot\frac{\pi}{4} = 2\pi.$$

$2\int_0^1 \sqrt{1-x^2}\, dx$

$$\overset{x=\sin\theta}{=\!=} 2\int_0^{\frac{\pi}{2}} \cos\theta\, d\sin\theta$$

$$= 2\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 2\cdot\frac{\pi}{4} = \frac{\pi}{2}$$

So the answer is $2\pi - \frac{\pi}{2} = \frac{3}{2}\pi.$