

Discussion – Tuesday, November 30th
Surface integrals of vector fields and related theorems

1. Consider the region D in \mathbb{R}^3 bounded by the xy -plane and the surface $x^2 + y^2 + z = 1$.
 - (a) Make a sketch of D .
 - (b) The boundary of D , denoted ∂D , has two parts: the curved top S_1 and the flat bottom S_2 . Parameterize S_1 and calculate the flux of $\mathbf{F} = (0, 0, z)$ through S_1 with respect to the upward pointing unit normal vector field. Check your answer with the instructor.
 - (c) Without doing the full calculation, determine the flux of \mathbf{F} through S_2 with the downward pointing normals.
 - (d) Determine the flux of \mathbf{F} through ∂D with the outward pointing normals.
 - (e) Apply the Divergence Theorem and your answer in (d) to find the volume of D . Check your answer with the instructor.
2. Consider the vector field $\mathbf{F} = (-y, x, z)$.
 - (a) Compute $\text{curl } \mathbf{F}$.
 - (b) For the surface S_1 above, evaluate $\iint_{S_1} (\text{curl } \mathbf{F}) \cdot \mathbf{n} \, dA$.
 - (c) Check your answer in (b) using Stokes' Theorem.
3. If time remains:
 - (a) Check your answer in 1(e) by directly calculating the volume of D .
 - (b) Repeat 2 (b-c) for the surface S_2 and also for the surface ∂D . What exactly does 2(c) mean for the surface ∂D ?
 - (c) For the vector field $\mathbf{F} = (-y, x, z)$ from the second problem, compute $\text{div}(\text{curl } \mathbf{F})$. Now suppose $\mathbf{F} = (F_1, F_2, F_3)$ is an arbitrary vector field. Can you say anything about the function $\text{div}(\text{curl } \mathbf{F})$?