Field Axioms

(A0) (Existence of addition) Addition is a well-defined process which takes pairs of real numbers a and b and produces from them one single real number $a+b$.

(A1) (Associativity) If a, b, and c are real numbers, then

$$a+(b+c)=(a+b)+c.$$

(A2) (Additive identity) There is a real number 0 such that for all real numbers a, we have

$$a+0=a \quad \text{and} \quad 0+a=a.$$

(A3) (Additive inverse) For every real number a, there is a real number $-a$ such that

$$a+(-a)=0 \quad \text{and} \quad (-a)+a=0.$$

(A4) (Commutativity) If a and b are any real numbers, then

$$a+b=b+a.$$

(M0) (Existence of multiplication) Multiplication is a well-defined process which takes pairs of real numbers a and b and produces from them one single real number $a \cdot b$ (often written as ab).

(M1) (Associativity) If a, b, and c are any real numbers, then

$$a \cdot (b \cdot c)=(a \cdot b) \cdot c.$$

(M2) (Multiplicative identity) There is a real number 1 such that for all real numbers a, we have

$$a \cdot 1=a \quad \text{and} \quad 1 \cdot a=a.$$

(M3) (Multiplicative inverse) For every real number $a \neq 0$, there is a real number a^{-1} such that

$$a \cdot a^{-1}=1 \quad \text{and} \quad a^{-1} \cdot a=1.$$

(M4) (Commutativity) If a and b are any real numbers, then

$$ab=ba.$$

(D) (Distributive law) For all real numbers a, b, and c,

$$a \cdot (b+c)=a \cdot b+a \cdot c, \quad \text{and} \quad (b+c) \cdot a=b \cdot a+c \cdot a.$$

(Z) (Non-triviality) $0 \neq 1$.

Any set F which satisfies these properties (after substituting "element of F" for "real number") is called a **field**.