Math 347 – Homework assignment #6
posted October 10, 2008; due Friday, October 17, 2008

Assigned exercises

1. Problem 6.3.
2. Prove that for every integer n, the numbers $2n + 5$ and $3n + 7$ are always relatively prime.
3. Problem 6.17. *Hint:* Show that the set of common divisors of $a + b$ and $a - b$ is equal to the set of common divisors of $2a$ and $a - b$ and equal to the set of common divisors of $a + b$ and $2b$.
4. Problem 6.22.
5. Problem 6.28. *Hint:* Use Proposition 6.6.
6. Problem 6.31. *Hint for (b):* By the division algorithm you can express a in the form $3k$, $3k + 1$ or $3k + 2$, and similarly for b. Take cases.
7. Problem 6.37(a). *Hint:* Start by noticing that p divides $\binom{p}{k}k!(p-k)! = p!$; then use the extended version of Euclid's lemma (Proposition 6.7 in the text).
8. How many positive divisors does $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$ have? How many positive divisors does $2^2 3^3 5^5$ have? Justify your answers.
9. Problem 6.39.
10. Problem 6.40. *Hint:* This requires you to sum a geometric series; for a reminder of how this is done see, e.g., Exercise 3.35.

Extra credit

The following problem, if attempted, must be turned in **on a separate sheet of paper**.

(**EC**) Suppose that a and b are relatively prime positive integers. Define A as the set of positive divisors of a and B as the set of positive divisors of b. Let c be the product of a and b, and let C be the set of positive divisors of c. (For example, if $a = 4$ and $b = 3$, then $A = \{1, 2, 4\}$, $B = \{1, 3\}$, $c = 12$, and $C = \{1, 2, 3, 4, 6, 12\}$.)

(a) Prove that if $(d, e) \in A \times B$, then $de \in C$.

(b) Prove that for each $c \in C$, there is a unique ordered pair $(d, e) \in A \times B$ for which $de = c$.

(c) For each positive integer m, we write $\sigma(m)$ for the sum of the positive divisors of m. Thus, for example, $\sigma(4) = 1 + 2 + 4 = 7$ and $\sigma(3) = 1 + 3 = 4$. Use the results of parts (a) and (b) to show that for any pair of relatively prime positive integers a and b, we have $\sigma(ab) = \sigma(a)\sigma(b)$. Give an example showing that this last equality may fail if a and b are not relatively prime.