Math 347 – Homework #8
posted November 8, 2008; due TBA

Exercises

1. Exercise 13.6.

2. Exercise 13.11.

3. Exercise 13.23.

4. Exercise 13.28.

5. Exercise 13.30.

6. (The squeeze theorem) Let $\langle a \rangle$, $\langle b \rangle$, and $\langle c \rangle$ be sequences for which $a_n \leq b_n \leq c_n$ for all $n \in \mathbb{N}$. Prove that if $\lim a_n$ and $\lim c_n$ both exist and equal the same number A, then $\lim b_n = A$.

7. Exercise 13.25. You may assume that $\sqrt{x}$ exists for all positive real numbers x and that $\sqrt{x}$ is increasing on the set of positive real numbers.

8. Let S be a set of real numbers. We say S is *bounded below* if there is some real number M with the property that $x \geq M$ for all $x \in S$. In this case M is called a *lower bound* for S. (See Definition 13.3 in the text.) Use the completeness axiom (Axiom 13.4) to show that if S is bounded below, then S has a greatest lower bound. (The greatest lower bound of a set S is denoted $\inf S$; here inf is short for "infimum.")

9. Suppose a and b are irrational numbers with $a < b$. Prove that there is a rational number x with $a < x < b$.