

TEACHING PHILOSOPHY

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On the face of it, my most influential teacher seems an unlikely role model. Dr. Ross was elderly – already over ninety years old in 1998 when I attended his summer mathematics program for high school students. He had an archaic style of speaking and a penchant for convoluted metaphors. He was hard of hearing; he would ask students to correct his arithmetic mistakes (allegedly made on purpose to help him gauge students’ attentiveness), but sometimes not notice when they did. And it seemed like each lecture raised more questions than it answered. And yet when the summer of 1998 came to an end, I realized that over the course of his eight-week program, I had gone from a dabbler impressed by the magic of number theory to a full-fledged apprentice. Ever since then, I have seen the art of teaching as being aimed at transforming students from passive listeners to individuals capable of taking ownership of knowledge.

J. H. Conway has said that a mathematician is a magician who reveals his secrets. Arnold Ross revealed his teaching secrets in the ‘Prologue’, his own teaching philosophy statement. There he promotes the idea of teaching as the process of “setting the stage for telling flashes of awareness.” In the case of the Ross program, this stage-setting was facilitated by an intricate infrastructure. Ross’s lectures were supplemented by daily (Moore-method style) problem sets, and students at the program worked on these sets back in their Ohio State University dorm rooms, relying on their counselors (usually college students, also living in the dorms) to grade their sets and answer any questions. Well, not exactly any questions – in keeping with the spirit of the program, counselors were encouraged to allow the participants as much intellectual freedom as possible, including freedom to explore seeming dead-ends.

I garnered my first experience teaching as a rising freshman, when I returned to the Ross program for a second year, serving as a junior counselor (working with a single first-year student under the supervision of a counselor). I would return to the program for three of my college summers to serve as a counselor. (I missed one summer in order to attend an REU.) At the Ross program I earned a reputation for certain stock responses to students’ questions. A favorite was ‘well, what do you think?’, which I offered as an initial response to nearly any inquiry. As unhelpful as this reply seems, it not only revealed how much the student had thought about the problem, but each answer served as a challenge to figure out the minimal amount I could reveal that would put my student on the path to discovery. I became impressed with the power of the Socratic method, and my students always seemed grateful that I left enough to do for them to honestly stake a claim to their newfound knowledge.

As a graduate student, I have had the opportunity to teach two courses at Dartmouth: Math 3 (the accelerated, single-term first calculus course) and Math 6 (Finite Mathematics). The challenges of teaching a college class are rather different than those I faced in the somewhat idealized setting of the Ross program: Ross program students are often self-selected, and as a result highly motivated, which is not always the case for students in lower-level math classes. And college students have more to think about in their dorms than mathematics, with no counselors to keep them in line. Nevertheless, I have found that the principles I learned at the Ross program have remained an effective guide.

How does one set the stage for Ross’s “telling flashes of awareness” in an ordinary college classroom? As a first step, I attempt to appeal to their intellectual curiosity. For example, to initiate our study of graph theory in Math 6, I devoted twenty minutes to having my students attempt at the board to find a solution to the ‘gas-water-electricity’ problem. By the end of this, many of

them were convinced (without knowing the definitions!) that $K_{3,3}$ was a non-planar graph. This made it easier for them to understand both the definitions in graph theory and the motivation for considering the subject. At other times I purposely foster cognitive dissonance. For example, when it came to discussing unique factorization in Math 6, I made an effort to convince them that this fact, as engrained as it might be in their psyches, was much deeper than they might think. It was hard for them to come to terms with how something so obvious could be so hard to justify. To help them, we went through the steps of an argument that prime factorizations exist, and commented on how it did not seem similarly easy to show that every number had only *one* factorization.

Another way I attempt to encourage student discovery is to get them talking about mathematics with me. In both of my classes so far, I have emphasized repeatedly that students should not feel ashamed to seek me out in office hours. My encouragement was particularly effective in Math 3. Since I team-taught Math 3 with a group of seven other graduate students, the syllabus, the homework assignments, and even the schedule of which sections to cover each day, were all decided with minimal input from me. But I did have complete control of my office hours, and they proved immensely popular. Rather than holding them in my office, we often worked in the spacious outside alcove. And often our working extended past the assigned time-slot, as I would stay long to be sure I addressed all their questions. It was heartening to see students go out of their way on their evaluations to comment on my accessibility.

In addition to this, I encourage interaction within the classroom (both between students and with me). Many of the Math 6 classes were planned with this sort of interaction in mind. For example, rather than opening class with a discussion of the various formulas associated for counting permutations and combinations, I would put up a sample problem and ask for ideas on how we might solve it. This not only forced the students to put their thoughts into words, but I am convinced that the progression from concrete to abstract made the material more memorable; rather than memorizing an answer, they could refer back to an entire process.

I also believe that a teacher owes it to his class to inject something of himself into the material. One consequence is that I do not make any attempt to hide my sense of humor. (This is no doubt correlated with one Math 3 student not making any attempt to hide calling me a ‘bit of a dork’ on their evaluation.) More seriously, this belief impacts how I select topics when I teach. When designing a syllabus for a course, as I did with Math 6, I emphasize those topics that are most interesting to me, and within those topics, the particular examples that seem most appealing. When I taught Math 6, this manifested itself in my doing an entire segment on the foundations of number theory (greatest common divisors, unique factorization, basic modular arithmetic), requiring custom problem sets and original lectures, as this topic was omitted from our course textbook.

I am grateful for all my teaching experiences so far. Each such experience has made me not only a better educator, but also a better mathematician, casting new light on material I thought I had mastered. I am excited about the prospect of teaching in the future, and look forward to nudging more students towards the mathematical epiphanies they are destined for.