Formulas for Volume and Surface Area of Three-Dimensional Geometric Figures

1. Preliminaries

The following notation will be used:

r will be used to represent the length of the radius of either a circle or a sphere.

b will be used to represent the length of the base side of a polygon. In the case of a trapezoid, which has two bases that may be of different lengths, these will be denoted as b_1 and b_2.

B will be used to represent the area of a base face of a three-dimensional geometric figure.

h will be used to represent the length of the height of a geometric figure. Recall that the height of a geometric figure must be perpendicular to its base.

a will be used to represent the length of the apothem of a regular n-gon. An apothem is a line segment that extends from the center of a regular n-gon to one of its edges and is perpendicular to the edge that it intersects. Remember that you may need to use the Pythagorean theorem to find a.

P will be used to represent the perimeter of a regular n-gon.

ℓ will be used to represent the length of the slant height of a pyramid or cone. For a pyramid, the slant height is the height of the (triangular) lateral faces. For a cone, the slant height is a line segment that extends from the apex to the edge of the circular base. Remember that you may need to use the Pythagorean theorem to find ℓ.

In order to use the later formulas, you need to be able to find the area of two-dimensional geometric figures. The ones that occur most often are:

shape	area formula
circle	πr^2
triangle	$\frac{1}{2}bh$
square	b^2
rectangle	bh
parallelogram	bh
trapezoid	$\frac{1}{2}(b_1 + b_2)h$
regular n-gon	$\frac{1}{2}aP$

2. Volume

For prisms and cylinders, the formula for volume is given by:

$$V = Bh$$

For pyramids and cones, the formula for volume is given by:

$$V = \frac{1}{3}Bh$$

The formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

3. Surface Area

The surface area of prisms and pyramids can easily be found by finding the area of each face and adding them together. For example, for a square pyramid, the area of the base, which is a square, is b^2, and the area of each lateral face is $\frac{1}{2}b\ell$. Thus, the surface area of a square pyramid is $b^2 + 4\left(\frac{1}{2}b\ell\right) = b^2 + 2b\ell$. As another example, the area of each face of a cube is b^2, and since a cube has six faces, its surface area is $6b^2$. Therefore, for the surface area of prisms and pyramids, it is probably easiest to just remember the concept of finding the area of each of the faces and adding them together rather than memorizing a separate formula for each figure.

A similar concept works for finding the surface area of a cylinder. Imagine trying to find the surface area of a soup can (which is a cylinder). You could use a can opener to cut off the two circles, and you could cut the soup label straight down and unroll it as a rectangle. The area of each of the circles is πr^2, and since the rectangle has a width that is equal to the circumference of the circle (which is $2\pi r$) and a height that is equal to the height of the cylinder, its area is $2\pi rh$. Thus, the formula for the surface area of a cylinder is:

$$\text{S.A.} = 2\pi r^2 + 2\pi rh$$

There are only a few figures for which it is definitely beneficial to have the formula for its surface area memorized:

- The formula for the surface area of a cone is:

  $$\text{S.A.} = \pi r^2 + \pi r\ell$$

- The formula for the surface area of a sphere is:

  $$\text{S.A.} = 4\pi r^2$$

- For the surface area of a hemisphere, we use half the surface area of the sphere for the round part (the yellow portion of the hemisphere from lab), but *do not forget about the circle at the bottom.* This adds an extra πr^2 to the surface area. Thus, the formula for the surface area of a hemisphere is:

  $$\text{S.A.} = 3\pi r^2$$