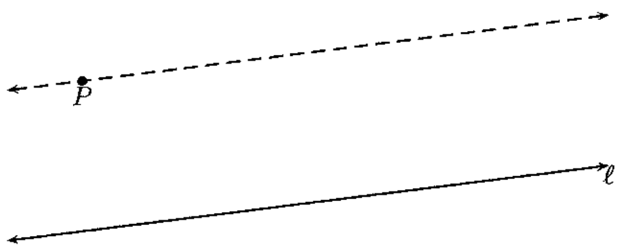
NOTES ON SPHERICAL GEOMETRY

First of all, a reminder about Euclidean geometry: For any line ℓ and any point P that is not on ℓ, there is exactly one line (drawn below dashed) passing through P that is parallel to ℓ. This fact is demonstrated below:



Now for some notes about spherical geometry. Two basic facts are:

- There are no parallel lines.
- Infinitely many lines go through antipodal points. (Think of the north and south poles and longitudes.) Otherwise, two points on the sphere determine a unique line.

Let π_E denote the value of pi in Euclidean geometry (approximately 3.14159) and Σ denote the angle sum of a spherical triangle (in degrees).
Now for some not so basic facts. Note that, in the following, the formulas that are numbered are the ones that we expect you to know.

- Σ is *not* constant! In fact, we have

(1) $$180^\circ < \Sigma < 540^\circ.$$

  Note that $\Sigma = 540^\circ$ occurs when the "triangle" is a great circle.
- The area of a triangle depends totally on its angle sum Σ. The area of a spherical triangle is

$$A = \frac{\pi_E}{180^\circ}\Sigma - \pi_E.$$

- In spherical geometry, pi is *not* constant! It depends totally on the radius of the circle. If r denotes the radius (in degrees) of a circle, then $\pi_S(r)$, the ratio of the circumference of the circle to its diameter, is given by

$$\pi_S(r) = \frac{180^\circ}{r}\sin\left(\frac{r\pi_E}{180^\circ}\right).$$

  You should be familiar with the inequality

(2) $$2 \leq \pi_S(r) < \pi_E.$$

  Note that $\pi_S(90^\circ) = 2$, meaning that pi equals 2 for a great circle.
- The area of a circle depends totally on its radius r, but the formula is no longer πr^2. The formula for the area of a circle in spherical geometry is

$$A = 2\pi_E(1 - \cos r).$$