KEY

Exam 1

Math 231

September 17, 2007

1. (a) (3 points) What derivative rule is the formula for integration by parts derived from?

Product Rule

(b) (8 points) Compute $\int \arctan x \, dx$.

$\int u\,dv = uv - \int v\,du$

$u = \arctan x \qquad dv = dx$

$du = \frac{1}{1+x^2}dx \qquad v = x$

$\int \arctan x = x\arctan x - \int \frac{x}{1+x^2}dx$

$= x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

2. (a) (3 points) For what integrals of the form $\int \sin^n x \cos^m x \, dx$ do you use the half angle formulas $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$?

When n and m are both even.

(b) (8 points) Compute $\int_0^\pi \sin^5 \theta \, d\theta$.

$$\int_0^\pi \sin^5 \theta \, d\theta = \int_0^\pi (\sin^2\theta)^2 \sin\theta \, d\theta$$

$$= \int_0^\pi \left(1 - 2\cos^2\theta + \cos^4\theta\right) \sin\theta \, d\theta$$

$$= -\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta \Big]_0^\pi$$

$$= -(-1) + \frac{2}{3}(-1) - \frac{1}{5}(-1) - \left(-1 + 2/3 - 1/5\right)$$

$$= 2\left(1 - 2/3 + 1/5\right) = 16/15$$

3. (a) (3 points) Explain why trigonometric substitutions help simplify certain integrals.

A) Trig subs eliminate + or - signs

B) Trig subs frequently eliminate radicals

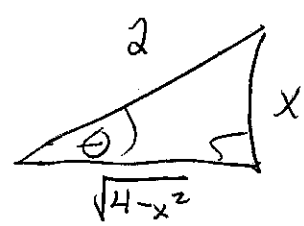
(b) (8 points) Compute $\int_0^1 \frac{1}{(4-x^2)^{3/2}}\,dx$.

$x = 2\sin\theta$ $\qquad dx = 2\cos\theta\, d\theta$

$$\int_0^1 \frac{1}{(4-x^2)^{3/2}}\,dx = \int \frac{2\cos\theta}{(4-4\sin^2\theta)^{3/2}}\,d\theta$$

$$= \int \frac{2\cos\theta}{(4\cos^2\theta)^{3/2}}\,d\theta$$

$$= \int \frac{2}{8}\sec^2\theta\,d\theta$$



$$= 1/4\tan\theta$$

$$= 1/4\left(\frac{x}{\sqrt{4-x^2}}\right)\Big]_0^1$$

$$= 1/4\,\frac{1}{\sqrt{3}} - 0 = \frac{1}{4\sqrt{3}}$$

4. (8 points) Use the method of partial fractions to evaluate $\int \frac{3x^3+3x^2+x+3}{x^2(x^2+1)}\,dx$.

$$\frac{3x^3+3x^2+x+3}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$3x^3+3x^2+x+3 = Ax^3+Ax+Bx^2+B+Cx^3+Dx^2$$

$$3 = A+C \quad \Rightarrow 2 = C$$
$$3 = B+D \quad \Rightarrow 0 = D$$
$$1 = A$$
$$3 = B$$

$$\int \frac{3x^3+3x^2+x+3}{x^2(x^2+1)}\,dx = \int\left(\frac{1}{x} + \frac{3}{x^2} + \frac{2x}{x^2+1}\right)dx$$

$$= \ln|x| - \frac{3}{x} + \ln|x^2+1| + C$$

5. (6 points) How many limits are required to do the integral $\int_0^3 \frac{1}{x^2 - x}\, dx$? Say for which values of x the limits are taken and what direction they are (if applicable).

$x^2 - x = 0$ at $x = 0$, $x = 1$

There are 3 limits, one at $x = 0$, 2 at $x = 1$

$\lim_{x \to 0^+}$

$\lim_{x \to 1^-}$

$\lim_{x \to 1^+}$

6. (8 points) Determine if $\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2}\,dx$ converges, and if it does evaluate it.

$$\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2}\,dx = \int_{-\infty}^{0} \frac{2x}{(x^2+1)^2}\,dx + \int_{0}^{\infty} \frac{2x}{(x^2+1)^2}\,dx$$

$$= \lim_{a \to -\infty} -\frac{1}{x^2+1}\Big]_a^0 + \lim_{b \to \infty} -\frac{1}{x^2+1}\Big]_0^b$$

$$= \lim_{a \to -\infty}\left(-\frac{1}{1} + \frac{1}{a^2+1}\right) + \lim_{b \to \infty}\left(\frac{-1}{b^2+1} + \frac{1}{1}\right)$$

$$= -1 + 0 + 0 + 1 = \boxed{0}$$