

Quiz 7

Math 124

April 3, 2009

NAME: KEY

1. You own an electronics company which makes laptops, desktops and phones. You have two factories, one in Chicago and one in St. Louis, and you have been monitoring the St. Louis factory for two weeks. In the first week the factory produces 100 laptops, 130 desktops and 212 phones. In the second week it produces 110 laptops, 120 desktops and 200 phones.

- (a) Write a production matrix for the two weeks at the St. Louis factory, and label the rows and columns.

$$\begin{matrix} & & \begin{matrix} L & D & P \end{matrix} \\ \begin{matrix} \text{Week 1} \\ \text{Week 2} \end{matrix} & \begin{bmatrix} 100 & 130 & 212 \\ 110 & 120 & 200 \end{bmatrix} \end{matrix}$$

- (b) You know that the factory in Chicago produces twice as many of each product as the St. Louis factory. Write a production matrix for the Chicago factory, labeling the rows and columns.

$$2 \begin{bmatrix} 100 & 130 & 212 \\ 110 & 120 & 200 \end{bmatrix} = \begin{bmatrix} 200 & 260 & 424 \\ 220 & 240 & 400 \end{bmatrix}$$

- (c) Determine how many of each product your company produces each week using matrix operations.

$$\begin{bmatrix} 100 & 130 & 212 \\ 110 & 120 & 200 \end{bmatrix} + \begin{bmatrix} 200 & 260 & 424 \\ 220 & 240 & 400 \end{bmatrix} = \begin{bmatrix} 300 & 390 & 636 \\ 330 & 360 & 600 \end{bmatrix}$$

(a) Compute the product AB of the matrices

3

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 \\ -2 & -2 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & -1 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ -2 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -6 \\ 18 & 7 \end{bmatrix}$$

(b) List, but DO NOT COMPUTE, all possible products among the following matrices.

2

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

AB AC BA CB

Note: Also CC , though
I didn't count this when
grading

Quiz 8

Math 124

April 10, 2009

NAME:

KEY

1. Find the inverse of the following matrix.

4

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 4 & 6 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

14:36

2. What is the essential, super-important, key equation for the Leontief open model with input-output matrix A and demand D ?

$$(I - A)^{-1} D = X$$

3. An economy is split into three sectors: Labor, Services and Manufacturing. Producing \$1 of Labor requires \$0.20 of Labor, \$0.40 of Services and \$0.00 of Manufacturing. Producing \$1 of Services requires \$0.30 of Labor, \$0.10 of Services and \$0.20 of Manufacturing. Producing \$1 of Manufacturing requires \$0.40 of Labor, \$0.40 of Services and \$0.30 of Manufacturing.

(a) Set up in the input-output matrix corresponding to this economy and label the columns and rows.

$$\begin{array}{c}
 \\
 L \\
 S \\
 M
 \end{array}
 \begin{bmatrix}
 & L & S & M \\
 L & .2 & .3 & .4 \\
 S & .4 & .1 & .4 \\
 M & .0 & .2 & .3
 \end{bmatrix}$$

(b) Identify which sectors are profitable and which are not.

$$.2 + .4 = .6 < 1$$

$$.3 + .1 + .2 = .6 < 1$$

$$.4 + .4 + .3 = 1.1 > 1$$

Labor and services are profitable
 Manufacturing is not

(c) Is the economy productive? Justify your answer.

Row sums:

$$\begin{array}{l}
 .9 \\
 .9 \\
 .3
 \end{array}$$

all < 1 , so it is productive.

Quiz 9

Math 124

April 17, 2009

Name:

KEY

1. You decide to get started early and open a retirement account. You have \$10,000 to invest and there are three funds available to you. Fund 1 contains 10% bonds, 20% securities and 70% stock. Fund 2 contains 80% bonds, 15% securities and 5% stock. Fund 3 contains 20% bonds, 60% securities and 20% stocks. You want your money to be divided so that you have at most 25% in bonds, at most 25% in securities and at least 50% in stocks.

(a) How many variables are there in this linear programming problem? Label them.

1

$$\begin{aligned}x &= \text{dollars in fund 1} \\y &= \text{dollars in fund 2} \\z &= \text{dollars in fund 3}\end{aligned}$$

(b) Set up, BUT DO NOT SOLVE, the linear program.

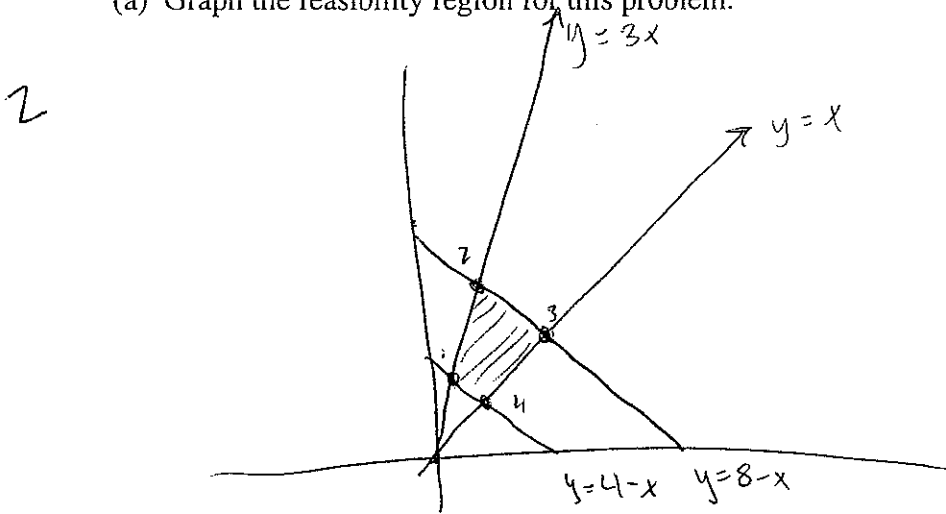
3

$$\begin{aligned}x + y + z &\leq 10,000 \\0.10x + .80y + .20z &\leq 2500 \\0.20x + .15y + .60z &\leq 2500 \\0.70x + .05y + .20z &\geq 5000 \\x &\geq 0 \\y &\geq 0 \\z &\geq 0\end{aligned}$$

2. Consider the following linear program:

$$\begin{aligned} \text{Maximize: } & 2x + 3y \\ \text{Subject to: } & y \leq 3x \\ & y \geq x \\ & y \geq 4 - x \\ & y \leq 8 - x \end{aligned}$$

(a) Graph the feasibility region for this problem.



(b) Find the optimum solution to this problem.

4

$$\begin{aligned} \text{PT 1: } & 3x = 4 - x \Rightarrow 4x = 4 \Rightarrow x = 1, y = 3 & (1, 3) \\ \text{PT 2: } & 3x = 8 - x \Rightarrow 4x = 8 \Rightarrow x = 2, y = 6 & (2, 6) \\ \text{PT 3: } & x = 8 - x \Rightarrow 2x = 8 \Rightarrow x = 4, y = 4 & (4, 4) \\ \text{PT 4: } & x = 4 - x \Rightarrow 2x = 4 \Rightarrow x = 2, y = 2 & (2, 2) \end{aligned}$$

PT	$2x + 3y$
(1, 3)	11
(2, 6)	22
(4, 4)	20
(2, 2)	10

← max

The maximum is 22 at (2, 6)

Quiz 10

Math 124

May 6, 2009

NAME:

KEY

1. You have \$1000 dollars invested in an account which earns 5% simple interest each year.

(a) State the formula which computes how much money you have after t years.

2

$$\begin{aligned} F &= P(1 + rt) \\ &= 1000(1 + .05t) \end{aligned}$$

(b) How much money do you have after 3 years?

2

$$\begin{aligned} F &= 1000(1 + .05 \cdot 3) \\ &= 1000(1.15) = \text{\$}1150 \end{aligned}$$

2. You want to buy a car, and you take out a \$15,000 loan to do so. You will make monthly payments over 5 years, and the interest on the loan compounds monthly at an annual rate of 12%.

(a) Is this an example of an annuity or a sinking fund?

Annuity

(b) How much interest does the bank earn on the loan in the first month?

$$i = \frac{.12}{12} = .01$$

$$\text{interest in first month} = .01 (15000) = \$150$$

(c) State the formula which computes your monthly payment, and fill in the given information in the appropriate places.

$$PMT = 15000 \frac{.01}{1 - (1.01)^{-60}}$$

(d) If your monthly payment works out to be \$333.67, what is the outstanding principal on the loan after the first month?

$$\begin{aligned} \text{Payment to principal} &= 333.67 - 150 \\ &= \$183.67 \end{aligned}$$

$$\begin{aligned} \text{Remaining principal} &= 15000 \\ &\quad - 183.67 \\ &\hline &= \$14816.33 \end{aligned}$$