Exam 2

Math 231

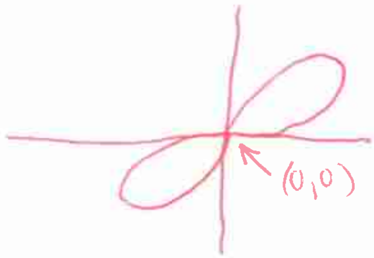
March 14, 2007

Name: KEY

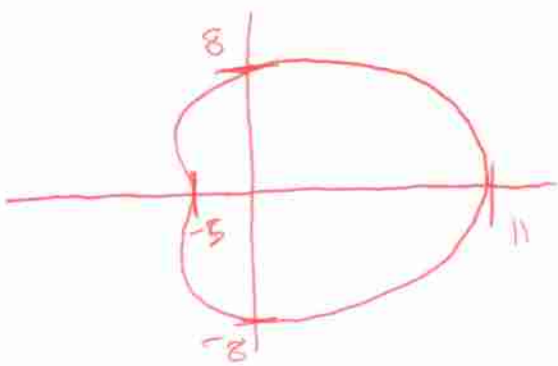
Directions

1. Be sure to show all work to receive full credit.
2. No calculators, books, notes, cell phones or ipods are allowed during the exam.
3. To receive full credit, you must state all rules that you use.
4. To receive full credit on questions that ask for graphs, label all points where the graph intersects the x- and y-axes. You may label these in either polar or rectangular coordinates.

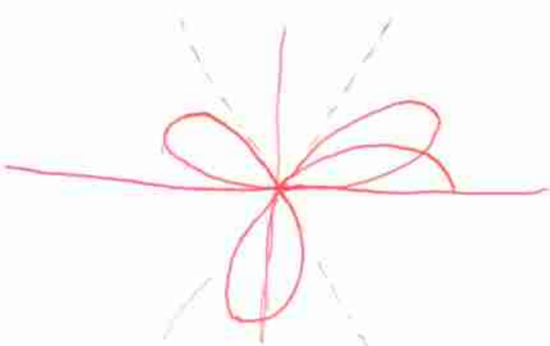
2. (a) (3 pts) Sketch the curve $r^2 = \sin 2\theta$ and label all points where it crosses the x- and y-axes.

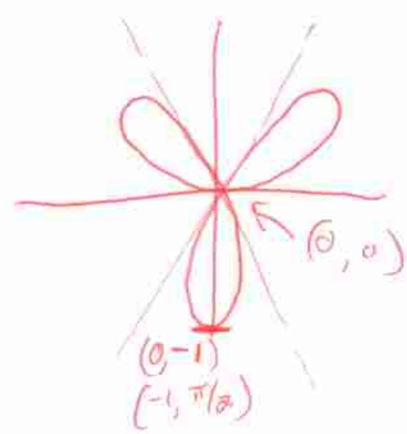


(b) (3 pts) Sketch the Curve $r = 8 + 3\cos\theta$ and label all points where it crosses the x- and y-axes.



(c) (3 pts) Sketch the curve $r = \sin 3\theta$.





(d) (3 pts) Convert the equation $r = \sin 3\theta$ into a set of parametric equations for x and y.

$x = \sin 3\theta \cos\theta$

$y = \sin 3\theta \sin\theta$

4. (a) (4 pts) Set up the integral for the surface area of the solid generated by revolving the curve $y = \sin t,\ x = t^2;\ 0 \le t \le 2\pi$ about the x-axis.

$$\int_0^{\pi} 2\pi \sin t \sqrt{(2t)^2 + \cos^2 t}\, dt - \int_{\pi}^{2\pi} 2\pi \sin t \sqrt{(2t)^2 + \cos t}\, dt$$

(b) (9 pts) Find the arc length of the curve $y = \sqrt{1-x^2},\ -1 \le x \le 1$. You may find the integral $\int \frac{1}{\sqrt{1-x^2}}\, dx = \arcsin x + C$ useful.

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$$

$$L = \int_{-1}^{1} \sqrt{1 + \left(\frac{-x}{\sqrt{1-x^2}}\right)^2}\, dx = \int_{-1}^{1} \sqrt{1 + \frac{x^2}{1-x^2}}\, dx = \int_{-1}^{1} \sqrt{\frac{1}{1-x^2}}$$

$$= \arcsin x \Big]_{-1}^{1} = \pi/2 - (-\pi/2) = \pi$$

6. (a) (4 pts) Set up the differential equation that models the natural growth equation in a city of population P with growth rate $\frac{1}{57}$.

$$\frac{dP}{dt} = \frac{1}{57} P$$

(b) (5 pts) Find the general solution of the differential equation in part (a).

$$\frac{dP}{P} = 1/57 \, dt$$

$$\ln p = \frac{1}{57} t + C$$

$$P = e^{1/57 t + C}$$

$$P = P_0 e^{1/57 t}$$

(c) (3 pts) What is the initial population of a city if it has a population of 600,000 people $57\ln(3)$ years from now?

$$600{,}000 = P_0 e^{1/57(57 \ln 3)}$$

$$= P_0 e^{\ln 3}$$

$$= 3P_0$$

$$200{,}000 = P_0$$