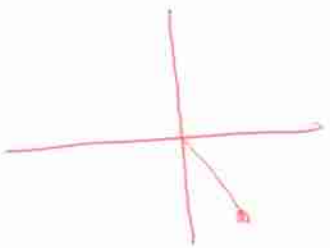
1. (a) (3 pts) Convert the point $(1, \sqrt{3})$ to polar coordinates.

$r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \pi/3$ $\quad (2, \pi/3)$

(b) (3 pts) Plot the point with polar coordinates $(-2, 2\pi/3)$ and convert to rectangular coordinates.



$x = -2\cos 2\pi/3 = 1/2$

$y = -2\sin 2\pi/3 = -\sqrt{3}/2$

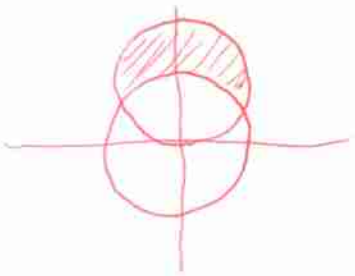
$(1/2, -\sqrt{3}/2)$

$(1, -\sqrt{3})$

(c) (3 pts) Convert the equation $y = x + 1$ to polar coordinates.

$r\sin\theta = r\cos\theta + 1$

3. (a) (5 pts) Set up but do not compute the integral that computes the area inside the circle $r = 5\sin\theta$ and outside the circle $r = \frac{5\sqrt{3}}{2}$.



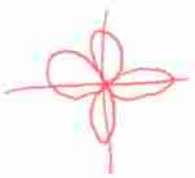
$5\sin\theta = 5\sqrt{3}/2$

$\sin\theta = \sqrt{3}/2$

$\theta = \pi/3,\ 2\pi/3$

$$\int_{\pi/3}^{2\pi/3} \frac{1}{2}\left(25\sin^2\theta - \frac{75}{4}\right) d\theta$$

(b) (9 pts) Find the area inside one leaf of the curve $r = 4\cos 2\theta$.



$0 = 4\cos 2\theta$

$0 = \cos 2\theta$

$\pm\frac{\pi}{2} = 2\theta$

$\pm\frac{\pi}{4} = \theta$

$$\int_{-\pi/4}^{\pi/4} \frac{1}{2}(16\cos^2 2\theta) = \int_{-\pi/4}^{\pi/4} 4(1+\cos 4\theta)\, d\theta$$

$$= 4\theta + \sin 4\theta \Big]_{-\pi/4}^{\pi/4} = \pi + \pi = 2\pi$$

5. (a) (4 pts) Set up but do not compute the integral for the area between the curve $y = 4\sin t,\ x = 4\cos t;\ 0 \le t \le \pi$ and the x-axis.

$$\int_{\pi}^{0} 4\sin t(-4\sin t)\,dt = \int_{\pi}^{0} -16\sin^2 t\,dt$$

(b) (9 pts) Find the volume of the solid generated by revolving the curve $y = 4\sin t,\ x = 4\cos t;\ 0 \le t \le \pi/2$ about the y-axis.

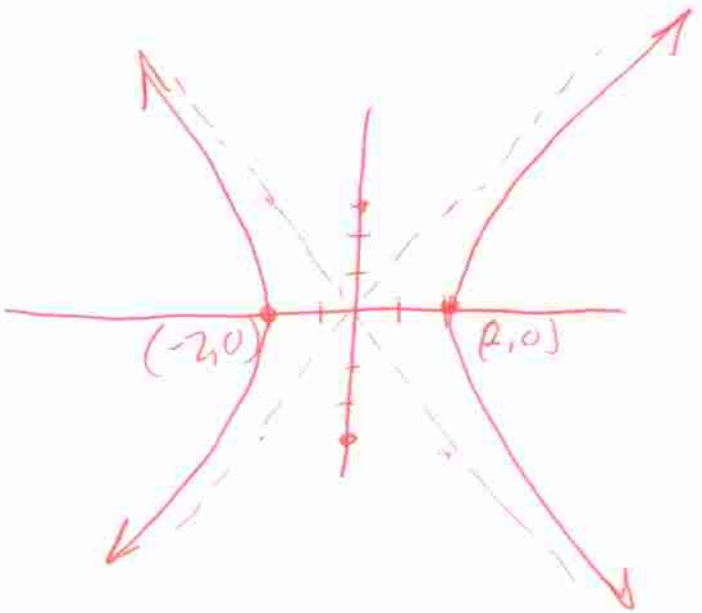
$$\int_{0}^{\pi/2} \pi(4\cos t)^2(4\cos t)\,dt = \int_{0}^{\pi/2} 64\pi\cos^3 t\,dt$$

$$= \int_{0}^{\pi/2} 64\pi(1-\sin^2 t)\cos t\,dt = 64\pi\sin t - \frac{64\pi}{3}\sin^3 t\,dt \Big]_{0}^{\pi/2}$$

$$= 64\pi - \frac{64\pi}{3} = \frac{128\pi}{3}$$

7. (a) (3 pts) Identify and sketch the conic section $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

~~Ellipse~~ Hyperbola



(b) (4 pts) Find the focus and the directrix of the parabola $x = 4y^2$.

$\frac{1}{4}x = y^2$

$4p = \frac{1}{4}$

$p = \frac{1}{16}$

Focus: $\left(\frac{1}{16}, 0\right)$

Directrix: $x = -\frac{1}{16}$