

# Final exam

Math 231

December 12, 2006

**Name:**

Directions

1. Be sure to show all work to receive full credit.
2. No calculators, books, notes or cell phones are allowed during the exam.
3. To receive full credit, you must state all rules that you use.
4. You may find the following integral useful.

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$$

1. (8 pts) Evaluate the integral  $\int_1^2 \ln x \, dx$ .

2. (a) (3 pts) State the half angle formula for  $\sin^2 x$ .

(b) (8 pts) Compute the integral  $\int \sec^4 x \, dx$ .

3. Determine if the following integrals converge or diverge.

(a) (6 pts)  $\int_0^4 \frac{1}{x^4} dx$

(b) (6 pts)  $\int_1^{\infty} \frac{1}{x^3} dx$

4. (a) (3 pts) State the comparison test.

(b) (6 pts) Determine if the series  $\sum_{n=1}^{\infty} \frac{x-1}{x^3+2}$  converges.

5. Consider the parametric curve  $x = 2 \cos \theta$ ,  $y = 2 \sin \theta$ .

(a) (4 pts) Convert the equation for the curve into rectangular coordinates and identify what type of conic section it is.

(b) (4 pts) Set up but do not compute the integral for the area that is inside the curve and to the right of the  $y$ -axis.

(c) (8 pts) Compute the volume of the solid that is generated by rotating the area in the previous part about the  $y$ -axis.

6. (a) (4 pts) Find the general form of the partial fraction decomposition of  $\frac{1}{(x-1)^2(x^2+1)}$ . Do not solve for  $A$ ,  $B$ , etc.

(b) (8 pts) Compute the integral  $\int \frac{x^3 + x^2 - x + 3}{x^4 + 2x^3 + 3x^2} dx$ .

7. (a) (4 pts) Write down the Maclaurin series for  $f(x) = e^{2x}$  and  $g(x) = \cos(x^3)$ .

(b) (6 pts) Using Maclaurin series (not L'Hopital's rule), compute  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ .

8. (8 pts) Find the third degree Taylor polynomial of  $f(x) = \ln(1 + x)$  at  $x = 3$ . There is no need to simplify the coefficients.

9. (a) (4 pts) Set up but do not compute the integral for the surface area of the solid that is generated by revolving the curve  $y = \sin x$ ;  $0 \leq x \leq \pi$  about the  $x$ -axis.

(b) (8 pts) Compute the surface area of the solid generated by revolving the curve  $x = 4t - 3$ ,  $y = 2t^2 - 3t + 5$ ;  $-1 \leq t \leq 1$  about the  $y$ -axis.

10. (a) (4 pts) Write two of the equivalent forms of the differential  $ds$  that we have used in this class.

(b) (9 pts) Find the length of the curve  $x = 2t$ ,  $y = t^2$ ;  $0 \leq t \leq 4$ .

11. Consider the series  $\sum_{n=1}^{\infty} \left(\frac{-1}{4}\right)^n$ .

(a) (6 pts) Prove that the series converges using the ratio test.

(b) (6 pts) Prove that the series converges using the root test.

(c) (6 pts) Evaluate the series.

12. (9 pts) Find the radius of convergence and the interval of convergence of the power series  $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^n}{\sqrt{n}}$ .

13. (a) (6 pts) Determine if the series  $a_n = \frac{n^2 + n - 1}{n^2 - 3}$  converges or diverges.

(b) (3 pts) State the bounded monotonic sequence property

14. (a) (3 pts) State the  $p$ -test for series.

(b) (10 pts) Use the integral test to determine the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  and  $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$ .