

Quiz 4 Solutions

Math 231

February 22, 2007

1. Set up but do not evaluate the following integrals.

(a) The arc length of the curve $x = \sin y$, $0 \leq y \leq \pi$.

SOLUTION: $\int_0^\pi \sqrt{1 + \cos^2 x} \, dx$

(b) The surface area of the solid generated by revolving the curve $y = x$, $-1 \leq x \leq 1$ about the x -axis.

SOLUTION: $\int_0^1 2\pi x \sqrt{1 + 1} \, dx - \int_{-1}^0 2\pi x \sqrt{1 + 1} \, dx$

2. Find the equation of the circle that has a diameter from $(1, 2)$ to $(11, 26)$

SOLUTION: Since the line segment connecting $(1, 2)$ and $(11, 26)$ is a diameter, the midpoint is the center of the circle. The midpoint is $(6, 14)$. Now to find the radius we find the distance between the points $(6, 14)$ and $(1, 2)$. This is $\sqrt{(6 - 1)^2 + (14 - 2)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. Thus the equation of the circle is

$$(x - 6)^2 + (y - 14)^2 = 13^2.$$

3. Find the arc length of the curve $y = \frac{2}{3}(x^2 + 1)^{3/2}$ from $x = 1$ to $x = 2$

SOLUTION: We use the integral for arc length. Notice that $\frac{dy}{dx} = (x^2 + 1)^{1/2}(2x)$.

$$\begin{aligned} L &= \int_1^2 \sqrt{1 + ((x^2 + 1)^{1/2}(2x))^2} \, dx \\ &= \int_1^2 \sqrt{1 + 4x^4 + 4x^2} \, dx \\ &= \int_1^2 \sqrt{(2x^2 + 1)^2} \, dx \\ &= \int_1^2 (2x^2 + 1) \, dx \\ &= \left. \frac{2}{3}x^3 + x \right|_1^2 \\ &= \frac{16}{3} + 2 - \frac{2}{3} - 1 = \frac{17}{3} \end{aligned}$$