

Quiz 9

Math 231

April 11, 2007

Name:

Be sure to show all work and state all tests that you use to receive full credit.

1. Determine the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^{7/2} + n + 2}$.

Solution 1: We use the comparison test. Note that $\frac{n^2 - 1}{n^{7/2} + n + 2} \leq \frac{n^2}{n^{7/2} + n + 2} \leq \frac{n^2}{n^{7/2}} = \frac{1}{n^{3/2}}$.

We know that $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by the p -test, so thus the series $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^{7/2} + n + 2}$ also converges.

Solution 2: We use the limit comparison test. We will compare the series to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$, which we know converges by the p -test.

$$\lim_{n \rightarrow \infty} \frac{\frac{n^2 - 1}{n^{7/2} + n + 2}}{\frac{1}{n^{3/2}}} = \lim_{n \rightarrow \infty} \frac{n^{7/2} - n^{3/2}}{n^{7/2} + n + 2}$$

It is easy to show that the limit is 1, so the series converges by the limit comparison test.

2. Use the ratio test to determine the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$. **Solution:** To use the ratio test we take the following limit.

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} \right| = \lim_{n \rightarrow \infty} \left| \frac{2}{n+1} \right| = 0$$

Since the limit is less than 1, the series converges by the ratio test.

3. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ is divergent, absolutely convergent, or conditionally convergent.

Solution: If we check the n th term test, the terms limit to $\ln 1 = 0$, so the test is inconclusive. We then check for absolute convergence. When we examine the series $\sum_{n=1}^{\infty} \frac{\ln n}{x}$ and compare it to $\sum_{n=1}^{\infty} \frac{1}{n}$, the comparison test shows us that the series diverges. Thus the series is not absolutely convergent. It remains to check for conditional convergence. We already know that $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$, and as the terms are decreasing, the alternating series test shows that the series is convergent. Thus the series is conditionally convergent.