# Merit Worksheet 23

Math 231

March 16, 2007

1. For the following lists, find the pattern and write down the formula for $a_n$, the nth term.

   (a) $2, 4, 6, 8, \ldots$

   (b) $1, 4, 9, 16, \ldots$

   (c) $1/2, 1/4, 1/8, 1/16, \ldots$

   (d) $1, 0, 1, 0, 1, 0, \ldots$

   (e) $1, x, x^2, x^3, x^4, \ldots$

   (f) $1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \ldots$

2. Look in the book and write down precisely what it means for a sequence to **converge** or **diverge**.

3. Given that $a_n \to 4$ and $b_n \to 6$, find the limit of the following sequences:

   (a) $\{a_n + b_n\}$

   (b) $\{5a_n b_n\}$

   (c) $\{\frac{b_n}{a_n}\}$

   (d) $\{a_n b_{n-1}$

   (e) When do the rules you just used not work?

4. For the following sequences, write down the first five terms. Then determine whether the sequences converge or diverge. If they converge, find the limit.

   (a) $a_n = \frac{1-n^2}{2+3n^2}$

   (b) $a_n = 2 - (-\frac{1}{2})^n$

   (c) $a_n = n \sin(\pi n)$

   (d) $a_n = \frac{\ln(2n)}{\ln(3n)}$

   (e) $a_n = \frac{\sin^2 n}{\sqrt{n}}$

   (f) $(-1)^n$

5. Use the squeeze law to evaluate the limit of the sequence $a_n = \dfrac{\cos n}{n}$.

6. Sequences can also be defined by recursion. Write down the first five terms of the following sequences:

(a) $a_1 = 2, a_n = a_{n-1} + 2$

(b) $a_1 = 4, a_n = a_{n-1}^{1/3}$

(c) $a_0 = 1, a_1 = 1, a_n = a_{n-1} + a_{n-2}$

7. Come up with two bounded monotonic sequences: one that increases and one that decreases.
8. Can an unbounded sequence converge?
9. Are all convergent sequences bounded?
10. Are all convergent sequences monotonic?
11. Can a bounded monotonic sequence diverge?
12. Find the limit of the following sequences:

(a) $\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\}$

(b) $\{\sqrt{20}, \sqrt{20 + \sqrt{20}}, \sqrt{20 + \sqrt{20 + \sqrt{20}}}, \ldots\}$